Outline

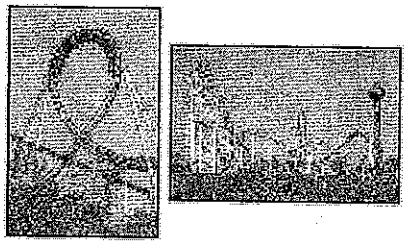
1. Functions defined by integrals.
2. Rules for manipulating integrals.

No recitation or lecture Thursday or Friday.

Handout 19: Who accelerates fastest of them all?

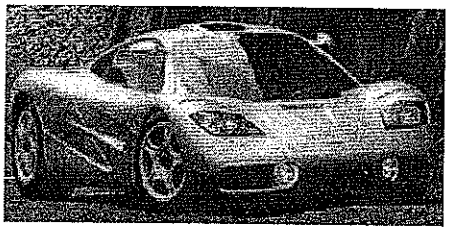
Figure 1 shows a variety of people, animals, weapons and vehicles. Everything and everyone pictured below can reach speeds in excess of 20 km per hour (equivalent to 13 miles per hour or 5.5 meters per second), and some can go a lot faster.

Donovan Bailey, a sprinter from Canada. Mr Bailey won two gold medals in the 1996 Olympic games and set a world record for the 100m.

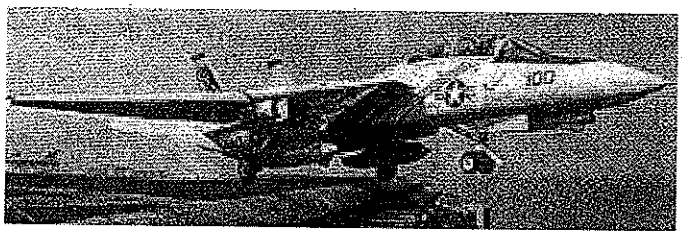


"The Viper." This roller coaster is one of the most popular rides at Six Flags Magic Mountain, located in Valencia, California. Pictured is the roller coaster's "clothoid loop" which is a section of the ride that subjects people to very powerful forces and accelerations.

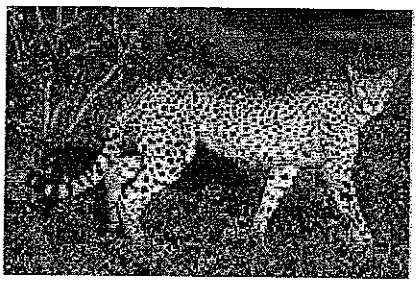
The Magnum .44 caliber revolver. The powerful handgun in the world. Why, it could blow your head clean off. The object of interest here is the bullet fired from the Magnum .44.



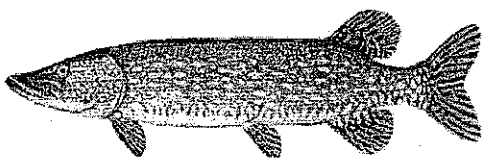
The McLaren F-1 GTR, the most expensive (and fastest) production car ever made.



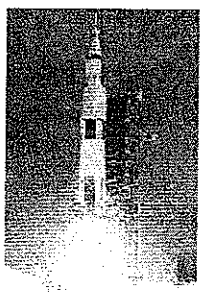
The F-14 Tomcat carrier launched fleet defence interceptor and air superiority fighter. Like other Navy jets, the F-14 uses a steam catapult to accelerate it when it launches from an aircraft carrier.



The cheetah (*Acinonyx jubatus*) is probably the fastest land animal over distances of a few hundred feet.



The Northern Pike (*Esox lucius)* is a ferocious freshwater predator found in North America. The pike grows to lengths of up to four feet and can weigh as much as 20 pounds. The pike is an "ambush predator" meaning that it prefers to lie in wait for prey items to wander by. The pike then lunges to capture its prey.



Apollo 11: The Saturn V rocket.

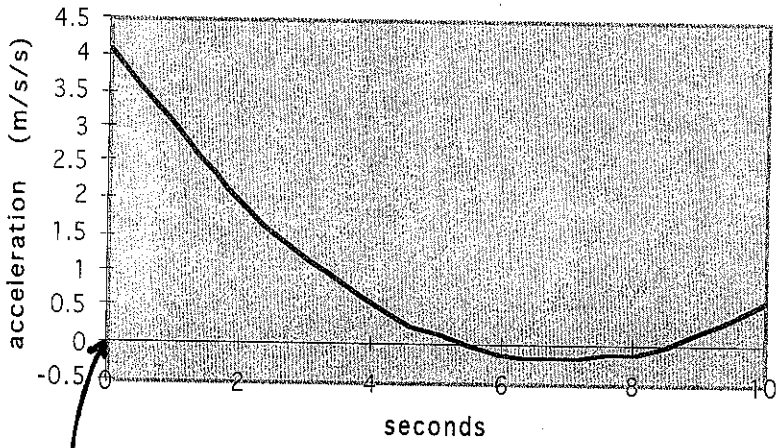
Figure 1: The people, animals, weapons and vehicles that you are supposed to rank.

1. Rank the creatures, people, weapons and vehicles according to the time they take to reach a speed of 20 km per hour. Record your rankings from 1 (i.e. gets to a speed of 20 km per hour the quickest) to 8 in Table 1 (below).

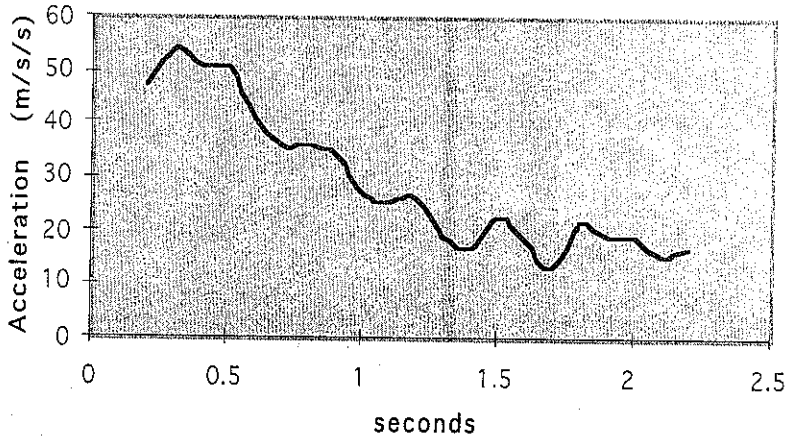
Person, creature, vehicle or weapon	Rank (1 to 8)
Donovan Bailey	
The "Viper"	
Magnum .44 bullet	
McLaren F-1 GTR	
Grumman F-14 Tomcat	
Cheetah	
Northern Pike	
Saturn V Rocket	

Table 1.

Donovan Bailey



The Viper

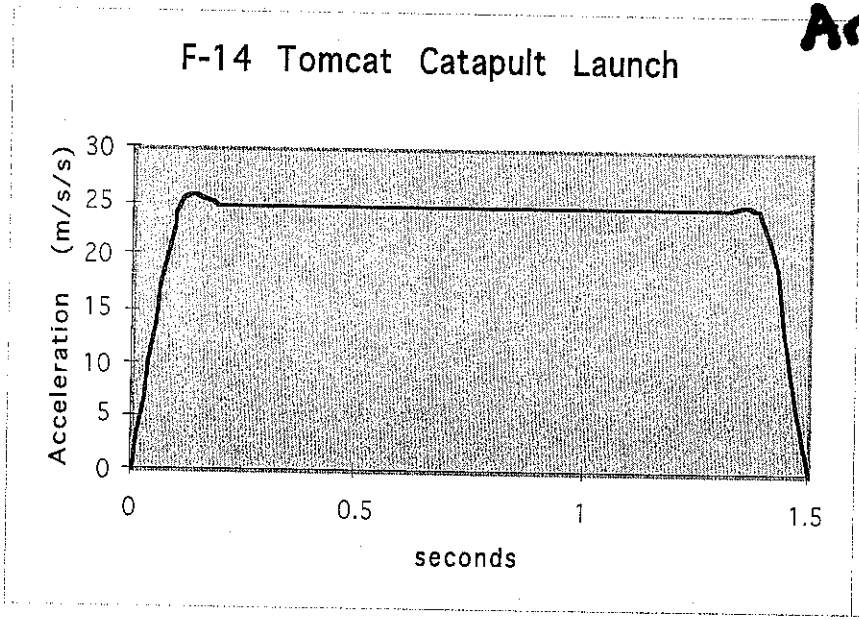


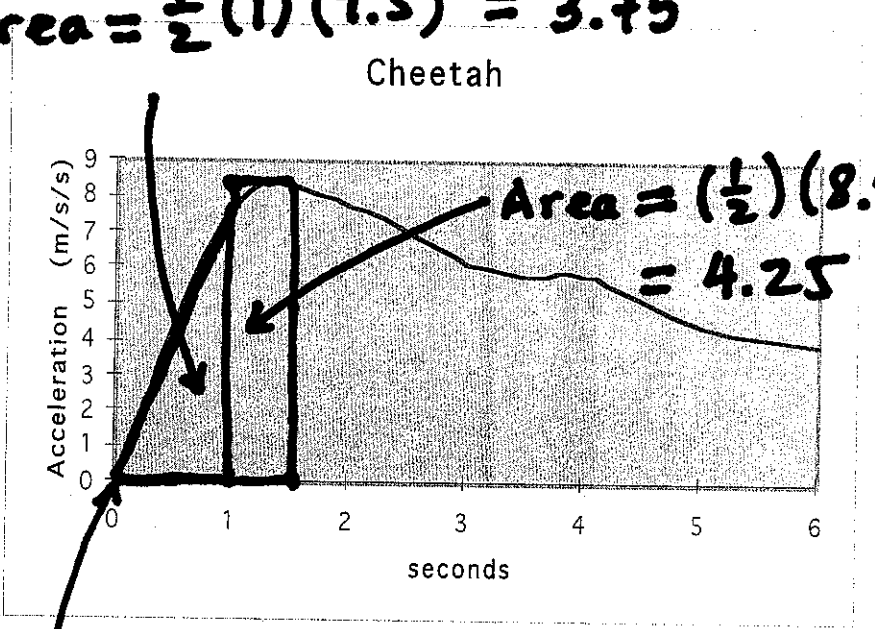
Speed = 0

Velocity

derivative ↓ ↗ Area under curve.

Acceleration





Area $= \frac{1}{2}(1)(7.5) = 3.75$

Area $= (\frac{1}{2})(8.5)$
$= 4.25$

Speed = 0

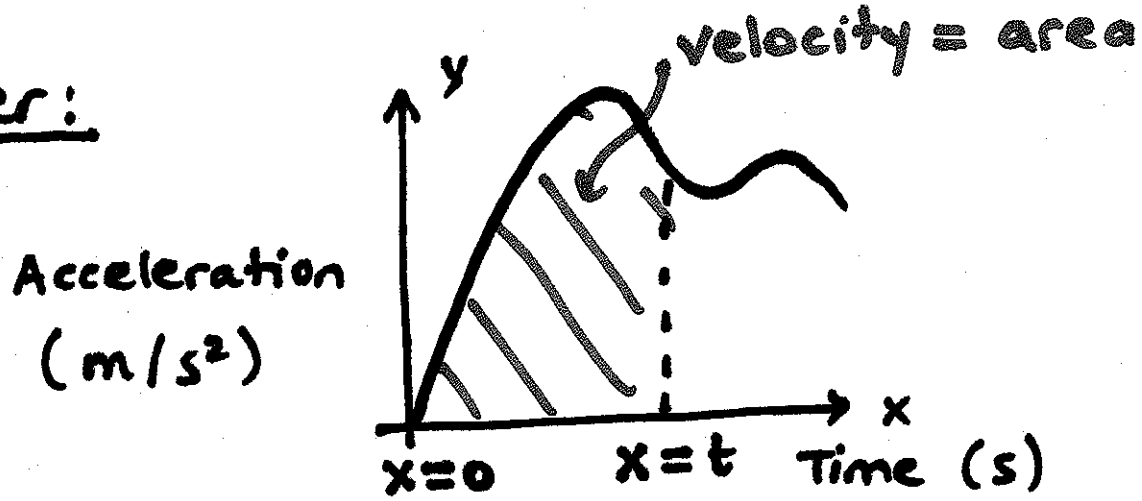
Area under graph = speed/or velocity.

Total area between $t=0$ and $t=1.5$ is

$\approx 3.75 + 4.25 = 8$ m/s.

1. Functions Defined by Integrals

- **Reminder:**



$a(t)$ = acceleration $v(t)$ = velocity

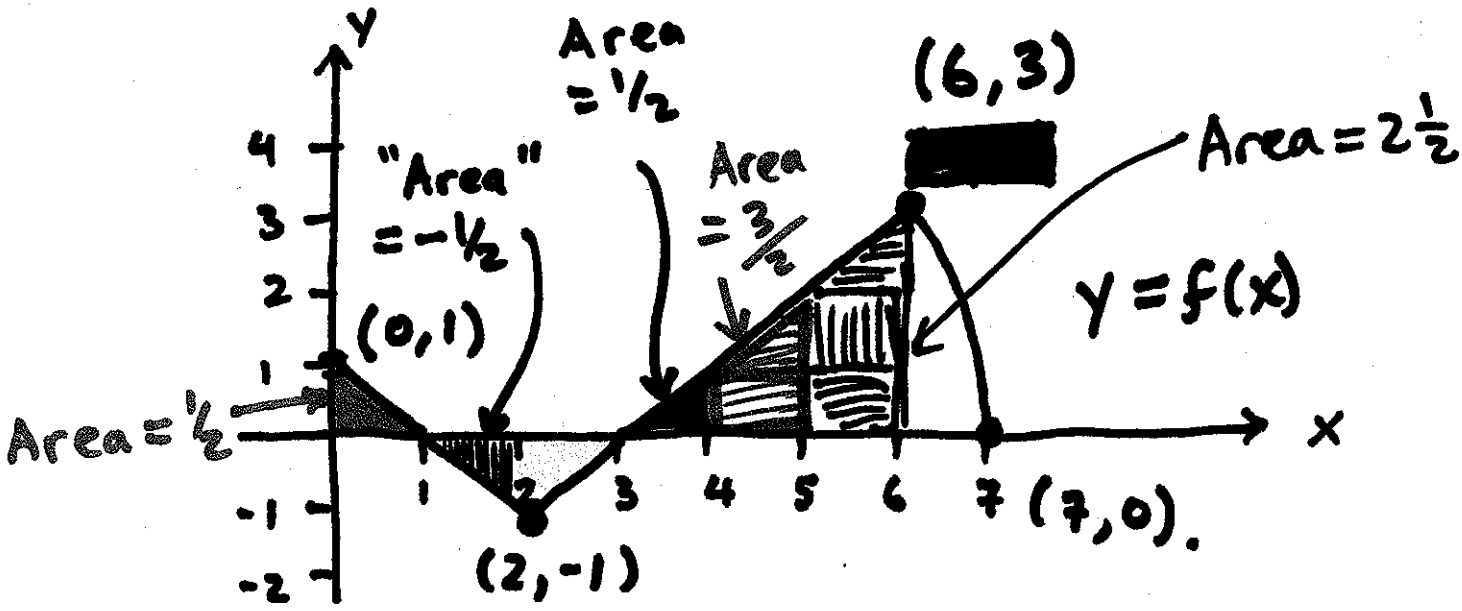
$$v(t) = \int_0^t a(x)\,dx$$

t ← the variable

x: can't do anything to or with x.

Example

Define $g(t) = \int_0^t f(x)\,dx$, where $f(x)$ is defined by the following graph:



(a) Find the values of $g(t)$ for $t = 0, 1, 2, 3, 4, 5, 6$.

(b) At what value of t does $g(t)$ attain its global max on $0 \leq t \leq 7$.

(c) Same question, but global min.

(d) Where are the points where $g'(t) = 0$?

Solution

(a)

t	0	1	2	3	4	5	6
g(t)	0	$\frac{1}{2}$	0	$-\frac{1}{2}$	0	$\frac{3}{2}$	4.

(b) $t = 7$ is where global max of $g(t)$ is attained. Most (positive) area above x-axis built up by $t = 7$.

(c) $t = 3$ is where global min of $g(t)$ is attained. Most (negative) area below x-axis is built up by $t = 3$.

(d) $$g(t) = \int_0^t f(x)\,dx$$

$$g'(t) = f(t) \cdot \frac{dt}{dt} - f(0) \cdot \frac{d0}{dt}$$

$$= f(t) \cdot 1 - f(0) \cdot 0$$

$$= f(t).$$

$g'(t) = 0$ when $f(t) = 0$. $t = 1, 3, 7$.