

Outline

1. Riemann sums.
2. Evaluating Riemann sums on a calculator.

—||—

Reminder: 1. Quiz Thursday.

2. Gateway test

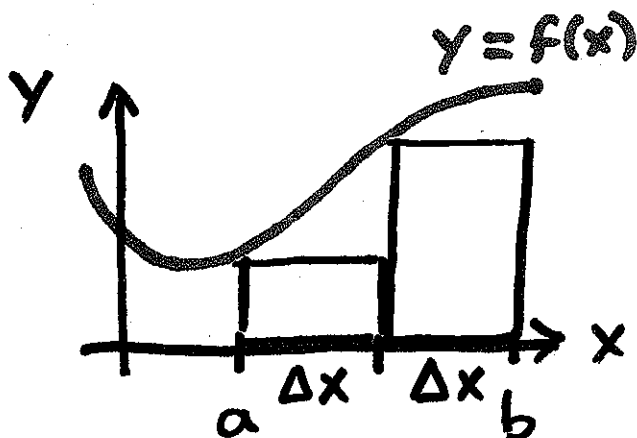
Deadline - 5/1/09.

1. Riemann Sums

- A way to approximate area under a curve using areas of rectangles.

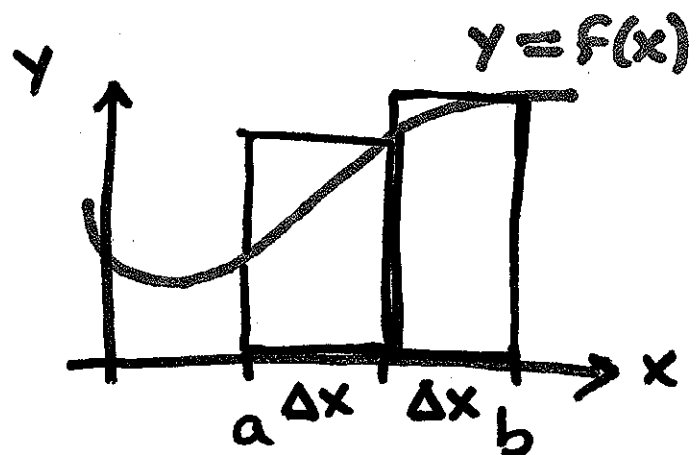
Left Hand Sums

- Height of rectangle is value of function on left edge of the rectangle's base.

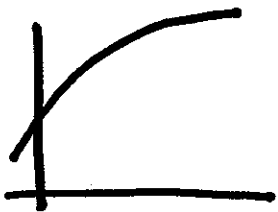


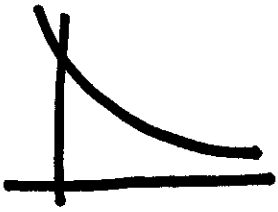
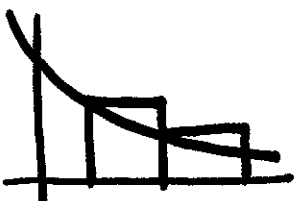



Right Hand Sums

- Height of rectangle is value of function on right edge of the rectangle's base.



- Sometimes the Riemann sums under or over estimate the "true" area under the curve.

Function	Left hand Sum	Right hand Sum
Increasing 	Underestimates true area 	Overestimates true area. 
Decreasing. 	Overestimates true area. 	Underestimates true area. 

- Estimates get more accurate when you use more rectangles.

Example

Estimate the area beneath

$f(x) = x^2$ between $a = 3$ and

$b = 7$ using 5 ($N = 5$) rectangles.

Solution

Calculate width of each rectangle.

$$\Delta x = \frac{b - a}{N} = \frac{7 - 3}{5} = 0.8$$

$$\begin{aligned} \text{Riemann Sum} &= \underbrace{f(3) \cdot 0.8}_{\text{area of rectangle 1}} + \underbrace{f(3.8) \cdot 0.8}_{\text{area of rectangle 2}} \\ &+ \underbrace{f(4.6) \cdot 0.8}_{\text{area of rectangle 3}} + \underbrace{f(5.4) \cdot 0.8}_{\text{area of rectangle 4}} \\ &+ f(6.2) \cdot 0.8 \} \text{ area of rectangle 5} \end{aligned}$$

$$= 89.76. \quad \text{Left hand Riemann sum.}$$

- The Right Hand sum is:

$$f(3.8) \cdot 0.8 + f(4.6) \cdot 0.8 + f(5.4) \cdot 0.8 \\ + f(6.2) \cdot 0.8 + f(7) \cdot 0.8 = 121.76.$$

- When a "best estimate" is asked for, average the left hand sum and right hand sum.