Outline

1. Linear functions.
2. Polynomials.
3. Sine and Cosine
4. Composite functions.

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① Class in session on Monday.

② HW #1 due on Tuesday.

Function Notation

$f(x)$ as the name of a function

"$f(x)$" as a noun

e.g. $f(x) = x^2 + 1$ the name of the formula is "$f(x)$" or "f"

"$f(x)$" as a verb

e.g. $f(1)$ means plug $x = 1$ into the formula and evaluate the result.

"$f(x)$" as a noun that looks like a verb

e.g. $\underbrace{f(1)}$ is the name of the quantity you get when $x=1$ is plugged into the formula.

always a "y-value"

Example

① $2 \cdot \underbrace{f(1)}_{\text{y-value on graph when } x=1\text{, i.e. } y=2} - \underbrace{g(0)}_{x=0 \text{ in the table, corresponding entry is } y=3} = (2)(2) - 3 = 1.$

y-value on graph when $x=1$, i.e. $y=2$

$x=0$ in the table, corresponding entry is $y=3$

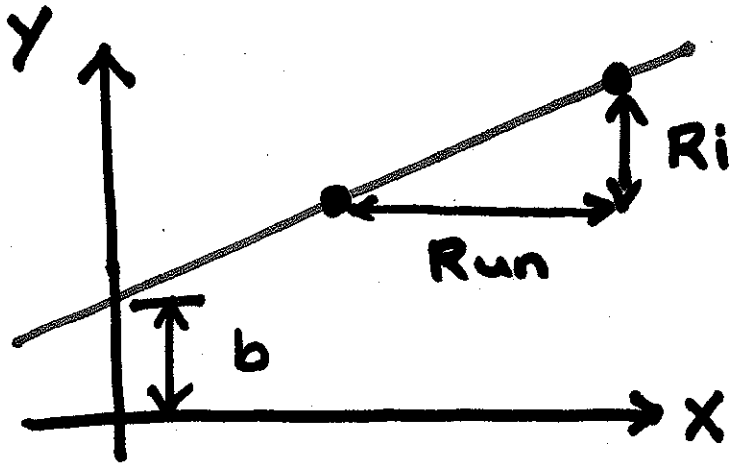
② $f(x) = 2$ Look where $y = 2$ on graph and find corresponding values of x.

$$x = 0, -1, 1.$$

③ $f(x) = g(x)$ Look where the graph & table have the same values of x & y. Solutions are x-values.

$$x = -1 \qquad x = 1$$

1. Linear Functions



$$m = \frac{\text{rise}}{\text{run}}$$

$$y = m \cdot x + b$$

$$m = \text{slope} = \frac{\Delta y}{\Delta x} = \frac{\text{change in } y}{\text{change in } x}$$

b = y-intercept = y-value when $x = 0$.

Example

Find a formula to convert Fahrenheit temperatures (F) to Celsius temperatures (C).

Solution

Phenomenon	Input F	Output C
Pure water freezes.	32	0
Pure water boils.	212	100

$$m = \frac{\Delta C}{\Delta F} = \frac{100 - 0}{212 - 32} = \frac{5}{9}$$

$$C = \frac{5}{9} F + \underbrace{b}_{\text{need to find}}$$

To get 'b' plug in one of the points. $(F, C) = (32, 0)$.

$$C = \frac{5}{9} F + b$$

plug in values

$$0 = \frac{5}{9}(32) + b$$

$$-\frac{5}{9}(32) = b$$

subtract $\frac{5}{9}(32)$ both sides

$$-17.77 = b$$

Final answer: $C = \frac{5}{9}F - 17.77$

2. Polynomial Function

- we will find formulas that look like:

numbers called multiplicities

$$f(x) = K \cdot (x - x_1)^{m_1} \cdot (x - x_2)^{m_2} \cdot \ldots \cdot (x - x_p)^{m_p}$$

a number, the constant of proportionality

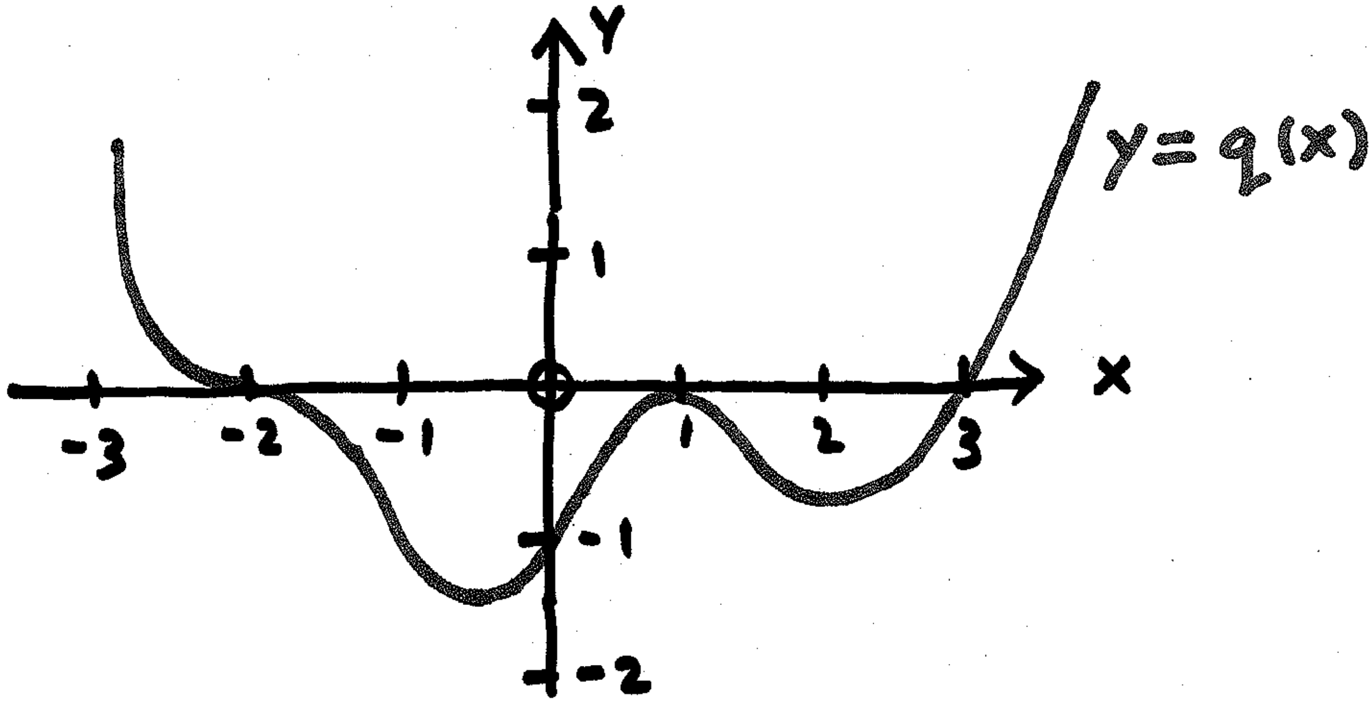
numbers, called roots or x-intercepts

- **Goal:** ① Find all the numbers, K, x_1, x_2, m_1, m_2, etc.

  ② Put together into a polynomial formula.

Example

Find the formula for $q(x)$ defined by:



Solution

① Find the roots (x-intercepts) of the polynomial.

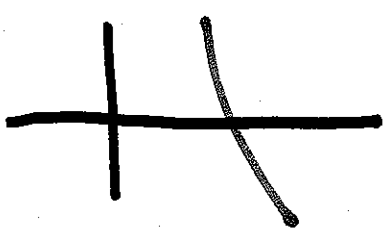
$x = -2$, $x = 1$, $x = 3$.

$x \in \{1, 3, -2\}$

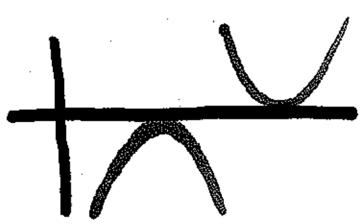
② Determine multiplicity of each root.

Root	Multiplicity
-2	3
1	2
3	1

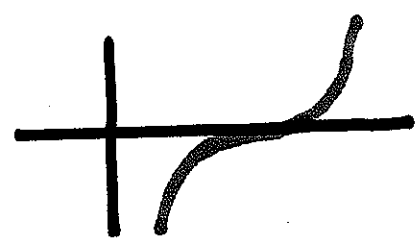
- Multiplicity is the shape that the graph makes as it passes through or touches the x-axis.



Multiplicity 1
"Clean Cut"



Multiplicity 2
"Tender caress"



Multiplicity 3
"John Travolta"

③ Write down a temporary formula.

$$q(x) = K \cdot (x - -2)^3 \cdot (x-1)^2 \cdot (x-3)^1$$

④ Determine value of k.

When $x = 0$, $y = -1$.

$$-1 = k \cdot (0+2)^3 \cdot (0-1)^2 \cdot (0-3)$$

$$-1 = k \cdot (-24)$$

$$\frac{1}{24} = k.$$

⑤ Write out the final answer.

$$q(x) = \frac{1}{24} \cdot (x+2)^3 \cdot (x-1)^2 \cdot (x-3)^1$$