Outline

1. Newton's method.
2. Concept of an anti-derivative.
3. Anti-derivative formulas.

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Reminder: Gateway deadline is 5/1/08 at 5pm.

1. Newton's Method

Main Formula: $$X_{n+1} = X_n - \frac{f(X_n)}{f'(X_n)}.$$

Example

Find a solution of the equation

$$\tfrac{1}{2}\cos(x) = x$$

accurate to 5 decimal places.

Solution

① f(x)=0 $\qquad f(x) = x - \frac{1}{2}\cos(x) = 0.$

② f'(x) $\qquad f'(x) = 1 + \frac{1}{2}\sin(x).$

③ Guess x_1 $\qquad X_1 = 0.$

④ $$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

n	x_n	$f(x_n)$	$f'(x_n)$	x_{n+1}
1	0	$-1/2$	1	$1/2$
2	$1/2$	0.0612087	1.2397127	0.4506267 (3dp match)
3	0.4506267	5.395336×10^{-4}	1.2177649	0.4501836 (5dp match)
4	0.4501836	4.65993×10^{-8}	1.21756542	0.4501836

- Since the two row's x_{n+1}'s match at least the first 5 decimal places, we can stop here and give:

$$x = 0.45018$$

as the solution of $\frac{1}{2}\cos(x) = x$.

2. The Concept of an Anti-Derivative

e.g. $f(x) = x^2$ DERIVATIVE

$F(x) = \frac{x^3}{3} + C$ (CONSTANT)

ORIGINAL FUNCTION THAT THE DERIVATIVE CAME FROM.

- If $f(x)$ is a function then $F(x)$ is an anti-derivative of $f(x)$ provided:

$$F'(x) = f(x)$$

Example

If $f(x) = e^{-x^2}$, [redacted]

sometimes, people think that:

$$F(x) = \frac{e^{-x^2+1}}{-x^2+1} + C$$

is an anti-derivative of $f(x)$.

Is it?

Solution

Plan:

1. Calculate $F'(x)$.
2. Simplify $F'(x)$ formula.
3. Check to see if the simplified formula is the same as the formula for $f(x)$.

$$F(x) = \frac{e^{-x^2+1}}{-x^2+1} + C$$

$$F'(x) = \frac{e^{-x^2+1} \cdot (-2x) \cdot (-x^2+1) - (-2x) e^{-x^2+1}}{(-x^2+1)^2}$$

$$= \frac{(-2x) e^{-x^2+1} \cdot (-x^2+1-1)}{(-x^2+1)^2}$$

$$= \frac{2x^3 \cdot e^{-x^2+1}}{(-x^2+1)^2}$$ Simplified derivative.

Is this the same as $f(x) = e^{-x^2}$?

Plug in $x=0$. $F'(0) = 0$

$f(0) = 1.$

Not equal, so $f(x) \neq F'(x)$.

So $F(x) = \dfrac{e^{-x^2+1}}{-x^2+1} + C$ is not an anti-derivative of $f(x) = e^{-x^2}$.

3. Anti-derivative Rules

$f(x)$	Anti-derivative
x^n $(n \neq -1)$	$\dfrac{x^{n+1}}{n+1} + C$
x^{-1} or $\frac{1}{x}$	$\ln(\lvert x \rvert) + C$
e^x	$e^x + C$
$\cos(x)$	$\sin(x) + C$

$\sin(x)$	$-\cos(x) + C$
$\sec^2(x)$	$\tan(x) + C$
$\sec(x)\tan(x)$	$\sec(x) + C$
$\frac{1}{\sqrt{1-x^2}}$	$\sin^{-1}(x) + C$
$\frac{1}{1+x^2}$	$\tan^{-1}(x) + C$