Outline

1. Optimization problem.
2. Newton's method.

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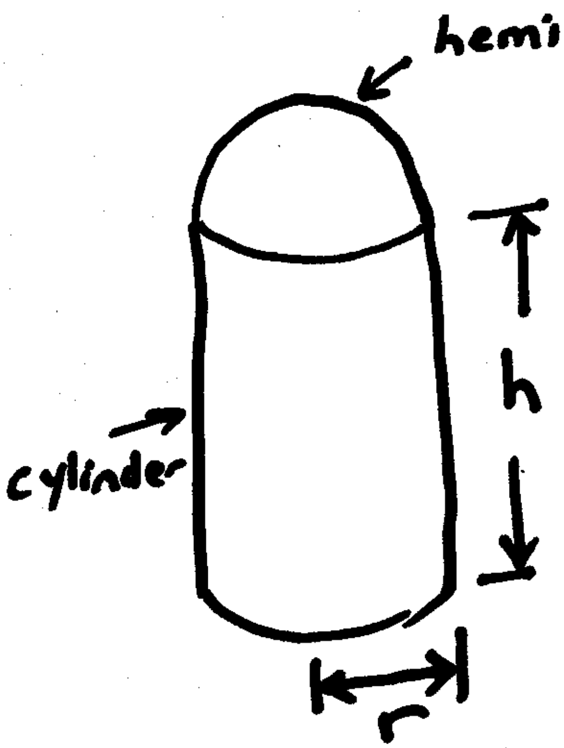
Do-over: Tonight
8-9pm or 9-10pm
1212 Doherty.

1. Optimization Problem

Example

Find the values of r, h that will give the least amount of surface area for the water bottle pictured if the volume is 500 cubic centimeters.



Solution

$$\text{Volume} = \underbrace{\pi r^2 h}_{\text{cylinder}} + \underbrace{\tfrac{2}{3}\pi r^3}_{\text{half a sphere.}}$$

$$\pi r^2 h + \frac{2}{3}\pi r^3 = 500$$

Solve for h:

$$\pi r^2 h = 500 - \frac{2}{3}\pi r^3$$

$$\boxed{h = \frac{500}{\pi r^2} - \frac{2}{3} r}$$

Surface area $= \underbrace{2\pi r \cdot h}_{\text{cylinder side.}} + \underbrace{2\pi r^2}_{\text{half a sphere}} + \underbrace{\pi r^2}_{\text{bottom of bottle}}$

Use $h = \frac{500}{\pi r^2} - \frac{2}{3} r$ to eliminate h:

$$S = 2\pi r\left(\frac{500}{\pi r^2} - \frac{2}{3} r\right) + 3\pi r^2$$

$$S = \frac{1000}{r} - \frac{4\pi}{3} r^2 + 3\pi r^2$$

$$S = \frac{1000}{r} + \frac{5\pi}{3} r^2$$

To find the critical points, solve $dS/dr = 0$.

$$\frac{dS}{dr} = \frac{-1000}{r^2} + \frac{10\pi}{3} r = 0$$

$$\frac{10\pi}{3} r = \frac{1000}{r^2}$$

$$\frac{10\pi}{3} r^3 = 1000$$

$$r^3 = \frac{300}{\pi}$$

$$r = \left(\frac{300}{\pi}\right)^{1/3} = 4.5707 \text{ cm.}$$

Second derivative test.

$$\frac{d^2S}{dr^2} = \frac{2000}{r^3} + \frac{10\pi}{3}.$$

Plug $r = 4.5707$ into this:

$$\frac{d^2S}{dr^2} = \frac{2000}{(4.5707)^3} + \frac{10\pi}{3} \approx 20.9439$$

positive.

Positive $\frac{d^2S}{dr^2}$ means $r = 4.5707$ gives a local minimum of surface area.

$$h = \frac{500}{\pi(4.5707)^2} - \frac{2}{3}(4.5707)$$

$$= 4.5711 \text{ cm.}$$

2. Newton's Method.

- Helps you find solutions of equations that are too hard to solve using algebra alone.

- If the ~~solution~~ equation we want to solve looks like:

$$f(x) = 0$$

then the only formula you need is:

$$\boxed{x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}}$$

- Steps for carrying out Newton's method.

① Rearrange the equation so it looks like:

$$f(x) = 0$$

② Calculate $f'(x)$.

③ Guess a solution, x_1, of the equation $f(x) = 0$.

④ (Repeats). Calculate:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

until you get the level of accuracy you need.