Outline

1. Curve sketching.
2. Max / Min problems.

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Remember: Do-over Wednesday
8 - 9 pm or 9-10 pm
1212 DH.

1. Curve Sketching

Example

Sketch: $y = f(x) = x^2 \cdot e^{+x}$.

Solution

A. Domain of Function.

- Domain is all real numbers.

B. Intercepts

- **x-intercept** Set $y = 0$

$$0 = x^2 \cdot e^x$$
$$0 = x^2$$
$$0 = x$$

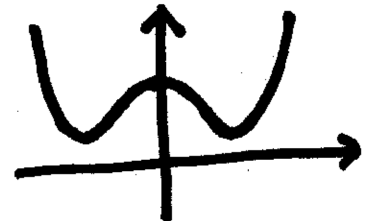
- **y-intercept** Set $x = 0$

$$y = 0^2 \cdot e^0 = 0.$$

Both intercepts occur at $(0,0)$.

C. Symmetry.

- <u>Even function</u> Need: $f(-x) = f(x)$



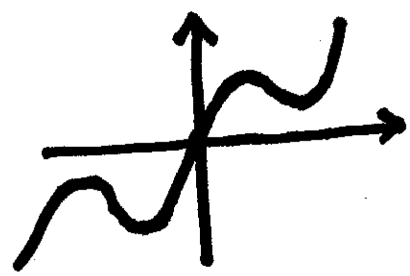
$$f(x) = x^2 \cdot e^x$$

$$f(-x) = (-x)^2 \cdot e^{-x} = x^2 \cdot e^{-x}$$

$$\neq x^2 \cdot e^x$$

So, $f(x)$ is not an even function.

- <u>Odd function</u> Need: $f(-x) = -f(x)$



$$f(-x) = x^2 \cdot e^{-x}$$

$$-f(x) = -\left(x^2 \cdot e^{+x}\right)$$

$$= -x^2 \cdot e^{+x} \neq x^2 \cdot e^{-x}$$

So, $f(x)$ is not an odd function.

D. Asymptotes.

- **Vertical asymptotes:** Need to find $x = a$ where at least one of:

$$\lim_{x \to a^-} f(x) = +\infty \qquad \lim_{x \to a^-} f(x) = -\infty$$

$$\lim_{x \to a^+} f(x) = +\infty \qquad \lim_{x \to a^+} f(x) = -\infty.$$

$$f(x) = x^2 \cdot e^x$$

The only way that $x^2 \cdot e^x \to \infty$ is when $x \to \infty$. So no vertical asymptotes.

- **Horizontal asymptotes:** Calculate:

$$\lim_{x \to \infty} f(x) \quad , \text{ and},$$

$$\lim_{x \to -\infty} f(x).$$

$$\lim_{x \to \infty} x^2 \cdot e^x = +\infty$$

So no horizontal asymptote as $x \to +\infty$.

$$\lim_{x \to -\infty} x^2 \cdot e^x = \lim_{x \to \infty} (-x)^2 \cdot e^{-x}$$

$$= \lim_{x \to \infty} \frac{x^2}{e^x} \qquad \frac{\infty}{\infty}$$

$$= \lim_{x \to \infty} \frac{2x}{e^x} \qquad \text{L'Hôpital}$$

$$= \lim_{x \to \infty} \frac{2}{e^x} \qquad \text{L'Hôpital}$$

$$= 0.$$

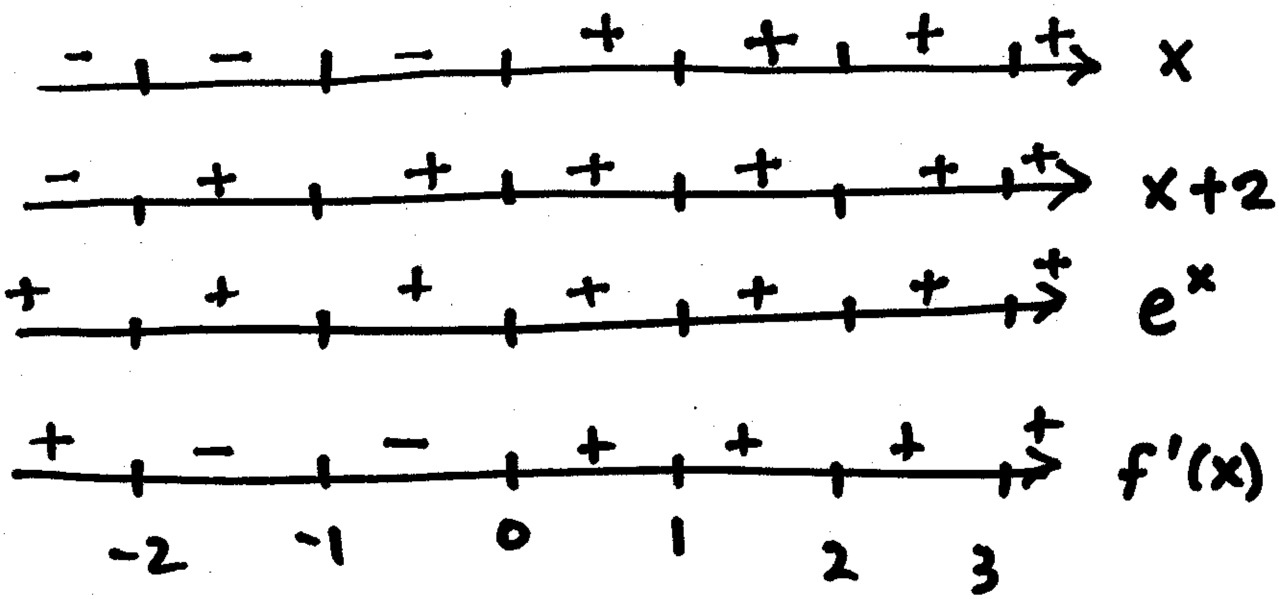
So, horizontal asymptote of $y = 0$ as $x \to -\infty$

E. Increasing / Decreasing.

$$f(x) = x^2 \cdot e^x$$

$$f'(x) = 2x \cdot e^x + x^2 \cdot e^x$$

$$= x \cdot (x+2) \cdot e^x$$



- $f(x)$ increasing when $x < -2$, $x > 0$

- $f(x)$ decreasing when $-2 < x < 0$.

F. Local mins and local maxes.

- Critical points: $f'(x) = 0$ or $f'(x)$ undefined

$x = -2$ $x = 0$

$+ \to -$ $- \to +$

Local max   Local min.

G. Concavity

$$f'(x) = (x^2 + 2x) \cdot e^x$$

$$f''(x) = (2x+2)e^x + (x^2+2x)e^x$$

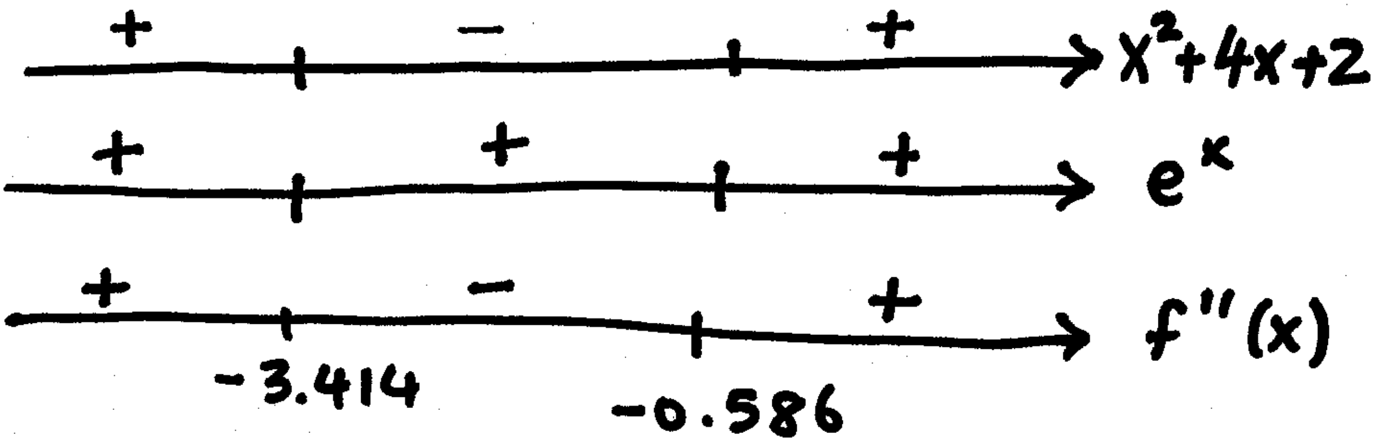
$$= (x^2 + 4x + 2)e^x$$

Solutions of $x^2 + 4x + 2 = 0$ are:

$$x = \frac{-4 \pm \sqrt{(4)^2 - (4)(2)}}{2} = -3.414 \text{ and } -0.586.$$

So $f''(x) = 0$ at $x = -3.414$

and $x = -0.586$



- $f(x)$ is concave up when $x < -3.414$ and when $x > -0.586$

- $f(x)$ is concave down when [redacted] $-3.414 < x < -0.586$.

- $f(x)$ has inflection points at $x = -3.414$ and $x = -0.586$.

Put all this together into a picture of the graph.