Outline

1. Finish the TV rebate example.
2. Inflection points.
3. Curve sketching.

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Do-over: Wednesday, April 8-9 pm or 9-10 pm 1212 Doherty.

1. TV Rebate Example

R = # of $10 rebates.

$$\text{\# of TVs sold} = 1000 + 100 \cdot R$$

$$\text{Price of TV} = 450 - 10R$$

$$\text{Revenue} = (1000 + 100R)(450 - 10R)$$

$$= 450\,000 + 35\,000R - 1000R^2$$

(a) Maximize revenue.

$$\text{Revenue} = 35\,000 - 2000R = 0$$

$$35\,000 = 2000R$$

$$\frac{35000}{2000} = R$$

$$17.5 = R$$

Offer $175 rebate to maximize revenue.

(b) Maximize profit.

x = # TVs sold

$C(x) = 68\,000 + 150x$

Need to make this a function of R. Substitute $x = 1000 + 100R$.

$$C(R) = 68000 + 150(1000 + 100R)$$

$$= 218\,000 + 15000R.$$

Profit = Revenue − Cost

$$= 450\,000 + 35000R$$

$$- 1000R^2 - 218000$$

$$- 15000R$$

$$\text{Profit} = 232\,000 + 20\,000R - 1000R^2$$

$$\text{Profit}' = 20\,000 - 1000 \cdot 2R = 0$$

$$20\,000 = 2000 \cdot R$$

$$\frac{20\,000}{2000} R$$

$$10 = R$$

Check to make sure profit is actually maximized with $R=10$:

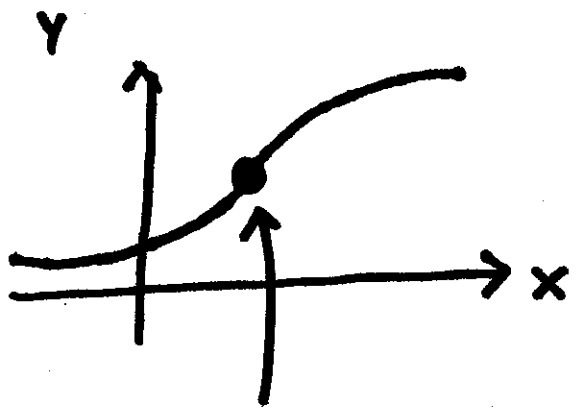
$$\text{Profit}'' = -2000 < 0.$$

Second Deriv.	Concavity	Max or Min?
$-$	Concave down $\cap$	Max.
$+$	Concave up $\cup$	Min

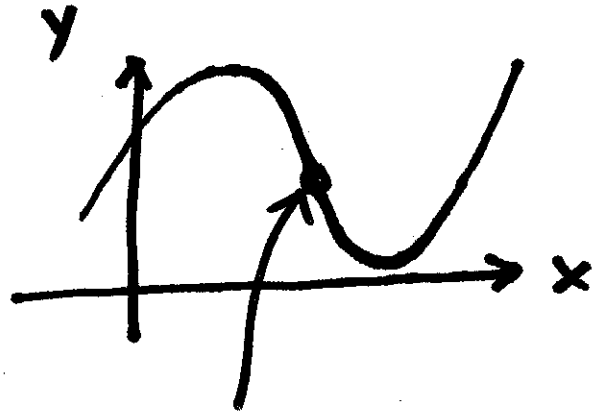
A \$100 rebate will maximize profits.

2. Inflection Points

- This is a point on the graph of a function where the concavity changes.



iflection point



inflection point.

- To find inflection points, find all points on $y = f(x)$ where:

  (a) $f''(x) = 0$

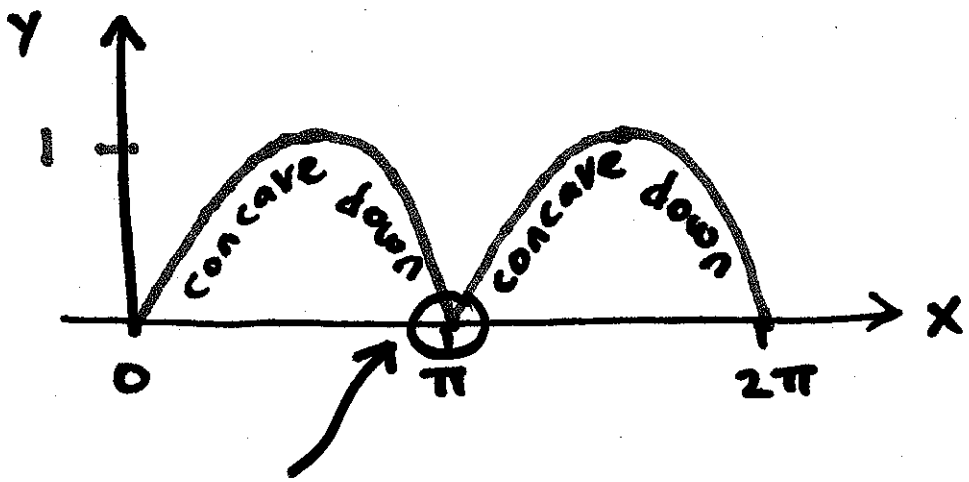
  (b) $f''(x)$ is undefined.

To check that there really is an inflection point there,

verify that $f''(x)$ changes sign (+ to − or − to +) from left side to the right side of the point.

Example

$$f(x) = |\sin(x)| \quad 0 \leq x \leq 2\pi.$$



at π we have a sharp corner. $f'(x)$ and $f''(x)$ both fail to be defined at $x = \pi$.

$x = \pi$ is not a point of inflection because the graph is concave down on both sides of $x = \pi$.

Example

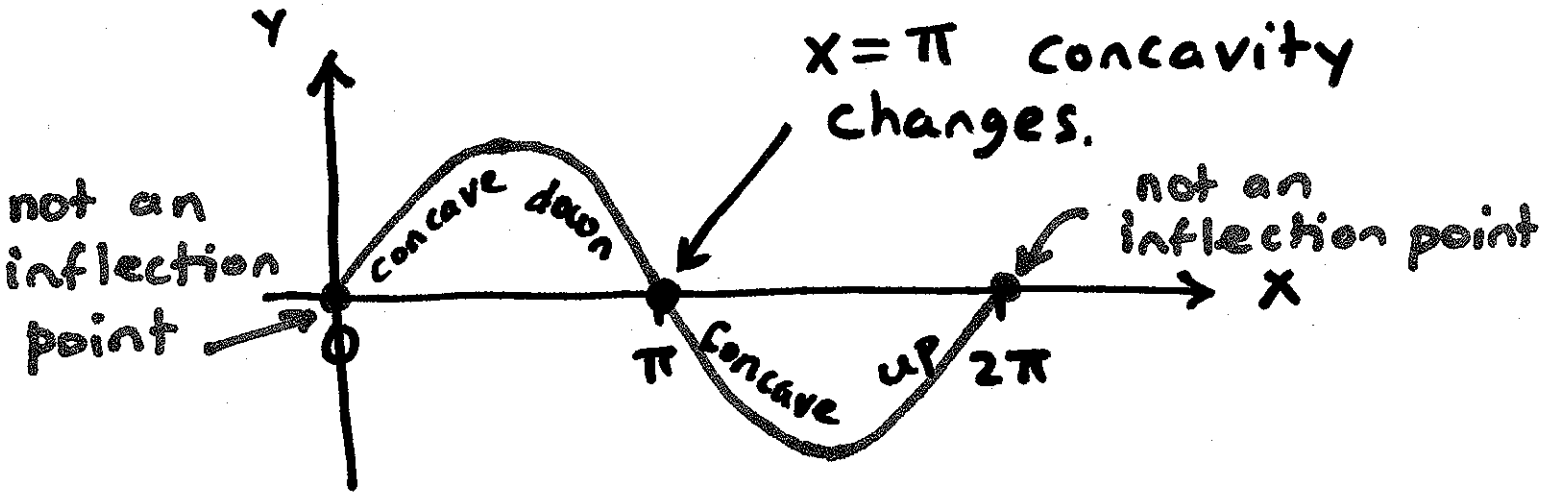
$f(x) = \sin(x) \qquad 0 \leq x \leq 2\pi$

$f'(x) = \cos(x)$

$f''(x) = -\sin(x) = 0$

Solutions: $x = 0 \quad x = \pi \quad x = 2\pi.$

$x = \pi$ this is an inflection point.



Example

$f(x) = x^4$

$f'(x) = 4x^3$

$f''(x) = 12x^2 = 0$

$x = 0.$

x	-0.1	0	0.1
$f''(x)$	0.12	0	0.12

no change in the sign of $f''(x)$, $x=0$ is not a point of inflection.

3. Curve sketching

- Things to pay attention to:
  - A. Domain.
  - B. x and y intercepts.
  - C. Symmetry (if any).
  - D. Asymptotes (horizontal & vertical).

E. Intervals where function is increasing / decreasing.

F. Local minimums, local maximums.

G. Points of inflection.

H. Intervals where graph is concave up or concave down.