Outline

1. Closed interval procedure.
2. First derivative test.
3. Second derivative test.

Do over: Wednesday 4/1
8-9 pm or 9-10pm

Drop deadline: Monday 3/30.

1. Closed Interval Procedure

- To find the global max and global min of a continuous function $f(x)$ on a closed interval $[a, b]$:

① Find all x-values for which $f'(x) = 0$.

② Find all x-values for which $f'(x)$ is undefined.

③ Find the end points of the interval $x = a$, $x = b$.

④ Evaluate $f(x)$ at all points found in ① - ③.

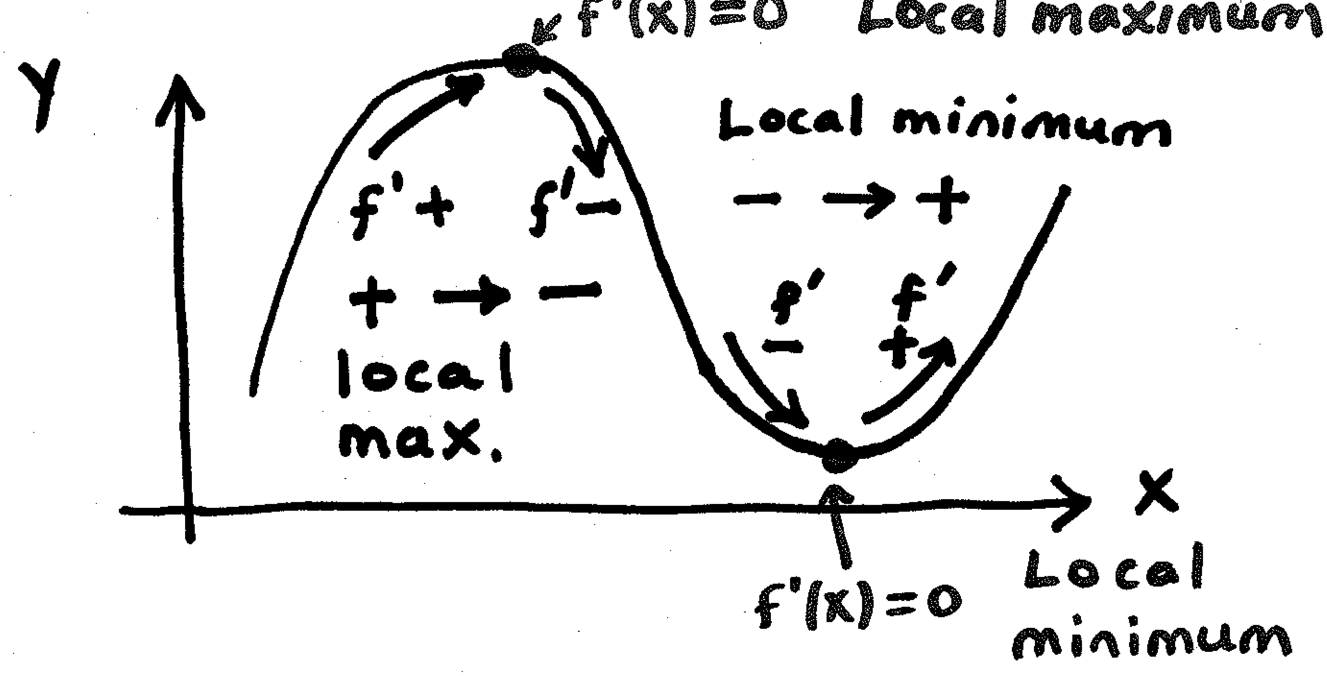
Global max = highest $f(x)$ in ④.

Global min = lowest $f(x)$ in ④.

2. First Derivative Test

First Derivative	Function	Graph
+	Increasing	
−	Decreasing	

- How this helps to classify maximums and minimums



Example

$$v(r) = k r^2 (r_0 - r)$$

$v(r)$: speed of air in the trachea

k: constant $k > 0$

r_0: radius of trachea under normal conditions $r_0 = 0.9 \text{ cm}$

r = radius of trachea when the person is coughing.

(a) Find values of r that make $v'(r) = 0$.

(b) Classify each point as a local max or local min.

Solution

$$v(r) = k r^2 (0.9 - r)$$

$$= k(0.9) r^2 - k r^3$$

(a) $$v'(r) = 2k(0.9)r - 3kr^2$$

$$= 1.8 k r - 3kr^2$$

Set $v'(r) = 0$ and solve for r.

$$1.8 k r - 3 k r^2 = 0$$

$$k \cdot r \cdot (1.8 - 3r) = 0$$

$\boxed{r = 0}$

$$1.8 - 3r = 0$$

$$1.8 = 3r$$

$$\boxed{\frac{1.8}{3} = r = 0.6}$$

(b) $\underline{r = 0}$

r	$r = -0.1$	$r = 0$	$r = +0.1$
$v'(r)$	$-0.21k$	0	$0.15k$

local minimum

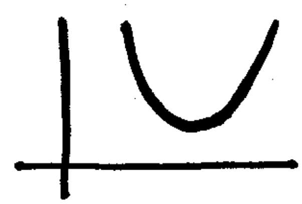
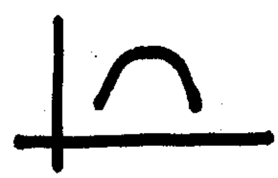
$r = 0.6$

r	$r = 0.5$	$r = 0.6$	$r = 0.7$
$v'(r)$	$0.15k$	0	$-0.21k$

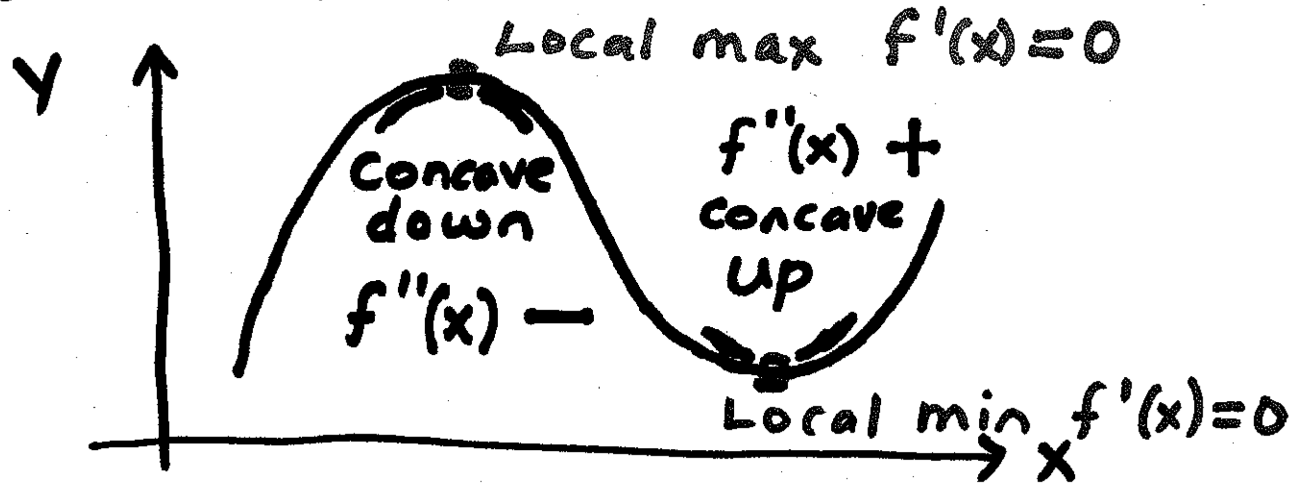
local maximum

2. Second Derivative Test

- $f''(x)$ tells you about the concavity of $y = f(x)$.

$f''(x)$	Function	Graph
+	Concave up	
−	Concave down.	

- $f''(x)$ can help distinguish local max from local min.



Example

A TV manufacturer sells 1000 TVs at a price of \$450. For every \$10 rebate, 100 more TVs are sold.

(a) To maximize revenue, how big of a rebate should be offered?

(b) If x TVs are sold, the cost is:

$$C(x) = 68\,000 + 150x.$$

How big of a rebate should be offered to maximize profit?

Solution:

Let R = # of of \$10 rebates given on each TV.

Price of TV $= 450 - 10 \cdot R$

Number of TVs Sold $= 1000 + 100 \cdot R$

$$\text{Revenue} = (\text{price})(\#\ \text{TVs sold})$$
$$= (450 - 10R)(1000 + 100R)$$
$$= 450\,000 + 35000R - 1000R^2$$