Outline

1. Definitions of global / local max and min.
2. Finding max and min values.
3. The closed interval method.

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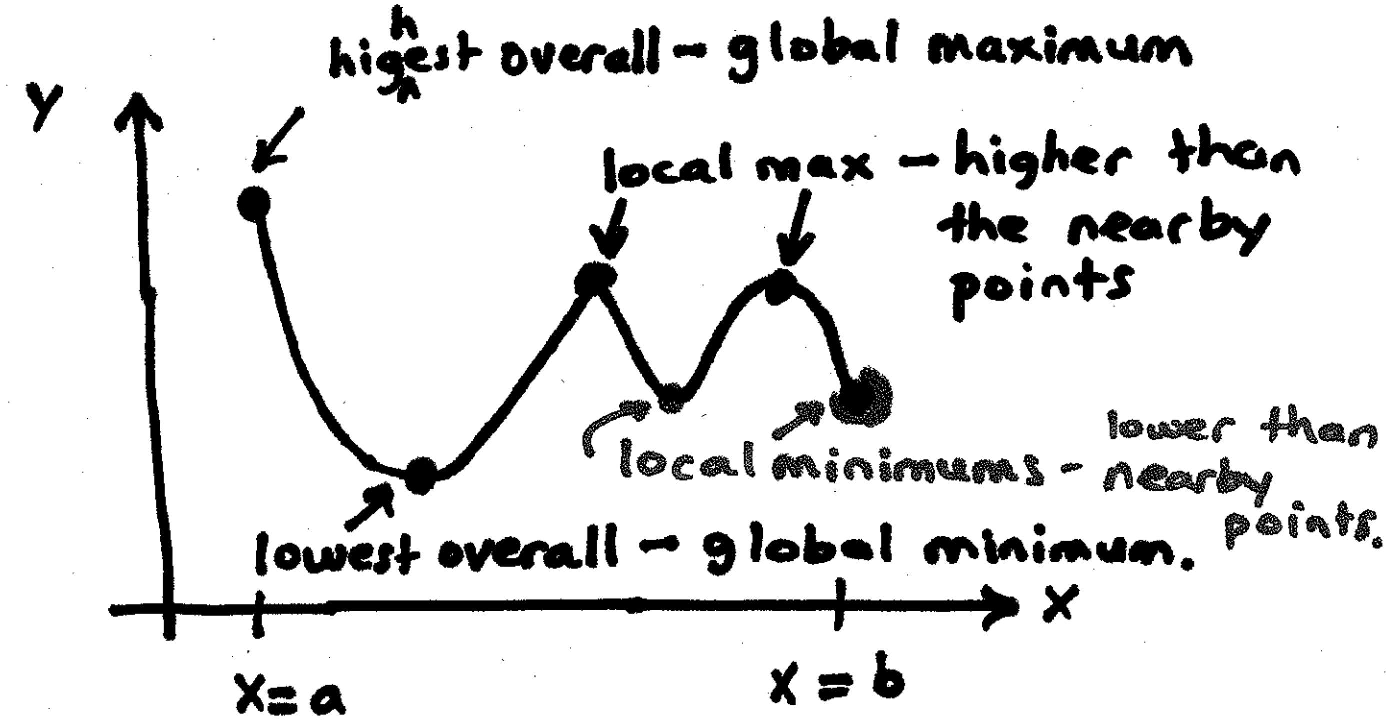
Do-over: Wednesday, April 1
8-9 pm or 9-10pm
1212 Doherty.

1. Global and Local Maximums and Minimums



- **Global max:** Highest possible point on graph between $x = a$ and $x = b$. (Also a local max.)

- **Global min:** Lowest possible point on graph between $x = a$ and $x = b$. (Also a local min.)

2. Points where Local Mins and Maxes Can Occur

- If we have a continuous function $f(x)$ and a closed interval of x-values we are interested in $[a,b]$ then the local mins and local maxes of $f(x)$ will occur at:

(a) Points where $f'(x) = 0$

(b) Points where $f'(x)$ is undefined.

(c) The endpoints of the interval $x = a$ and $x = b$.

- The global max is one of the

local maximums.

- The global min is one of the local minimums.

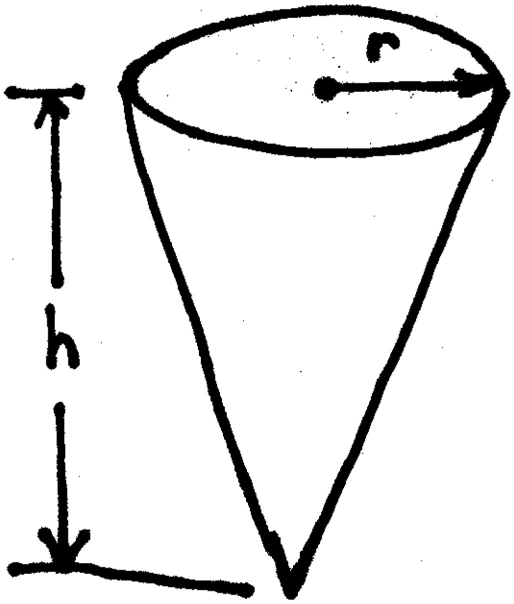
Example

The paper cup for a Sno-Cone™ is a cone that holds 8 oz of ice. (8 oz $\approx$ 24 cm^3.)

If we want to use as little paper as possible in the cone, what dimensions should the paper cup have?

Solution

We want to minimize surface area using the constraint that the volume has to be $V = 24\ cm^3$.



Surface area:

$$S = \pi r \sqrt{r^2 + h^2}$$

too many variables present - we can only take derivatives when the function has one variable.

<u>Strategy</u>: ① Write down an equation for the <u>constraint</u>

② Rearrange to make r, or h, the subject of that equation.

③ Replace one of the variables in the formula for S.

Volume of Cone $= \frac{1}{3} \pi r^2 h = 24.$

Make h the subject:

$$h = \frac{72}{\pi r^2}$$

Replace h in the formula for S with this:

$$S = \pi r \sqrt{r^2 + \left(\frac{72}{\pi r^2}\right)^2}$$

$$= \pi r \sqrt{r^2 + \frac{5184}{\pi^2 r^4}}$$

$$= \sqrt{\pi^2 r^4 + \frac{5184}{r^2}}$$

To find local max and local min, calculate $\frac{dS}{dr}$.

$$\frac{dS}{dr} = \frac{1}{2}\left(\pi^2 r^4 + \frac{5184}{r^2}\right)^{-1/2} \cdot \left(4\pi^2 r^3 - \frac{10368}{r^3}\right)$$

$$= \frac{4\pi^2 r^3 - \dfrac{10368}{r^3}}{2 \cdot \sqrt{\pi^2 r^4 + \dfrac{5184}{r^2}}}$$

First, set $\frac{dS}{dr} = 0$ and solve for r.

$\frac{dS}{dr} = 0$ $\qquad 4\pi^2 r^3 - \frac{10368}{r^3} = 0$

$$\frac{1}{r^3}\left(4\pi^2 r^6 - 10368\right) = 0$$

$r \neq 0$ $\qquad 4\pi^2 r^6 - 10368 = 0$

$$4\pi^2 r^6 = 10368$$

$$r^6 = \frac{10\,368}{4\pi^2}$$

$$r = \left(\frac{10368}{4\pi^2}\right)^{1/6} \approx 2.53 \text{ cm.}$$

To find h, plug $r = 2.53$

$$h = \frac{72}{\pi r^2} = 3.58 \text{ cm.}$$

Next, find values of r that make dS/dr undefined.

$$\frac{dS}{dr} = \frac{4\pi^2 r^3 - \dfrac{10368}{r^3}}{2\sqrt{\pi^2 r^4 - \dfrac{5184}{r^2}}}$$

Only point where dS/dr is undefined is $r = 0$.