1. L'Hôpital and Infinite Limits

- Can also use L'Hôpital's rule when limit looks like $\frac{\infty}{\infty}$.

__Example__

as $x \to 0^+$ $\ln(x) \to -\infty$

$$\lim_{x \to 0^+} \frac{\ln(x)}{\operatorname{cosec}(x)}$$

as $x \to 0^+$ $\operatorname{cosec}(x) \to +\infty$.

Can use L'Hôpital to solve this.

$$\frac{d}{dx}(\ln(x)) = \frac{1}{x}$$

$$\frac{d}{dx}(\operatorname{cosec}(x)) = \frac{d}{dx}\left(\frac{1}{\sin(x)}\right)$$

$$= \frac{0 \cdot \sin(x) - 1 \cdot \cos(x)}{\sin^2(x)}$$

$$= -\frac{\cos(x)}{\sin^2(x)}$$

So:

$$\lim_{x \to 0^+} \frac{\ln(x)}{\operatorname{cosec}(x)} = \lim_{x \to 0^+} \frac{\frac{1}{x}}{\frac{-\cos(x)}{\sin^2(x)}}$$

$$= \lim_{x \to 0^+} \frac{1}{x} \cdot \frac{-\sin(x) \cdot \sin(x)}{\cos(x)}$$

$$= \lim_{x \to 0^+} \frac{\sin(x)}{x} \cdot \lim_{x \to 0^+} \frac{-\sin(x)}{\cos(x)}$$

$$= (1)\left(\frac{0}{1}\right)$$

$$= 0.$$

Example

- Can use L'Hopital's rule to calculate limits as $x \to \infty$.

$$\lim_{x \to \infty} \frac{x^2+1}{x \cdot \ln(x)}$$

Solution

$$\lim_{x \to \infty} x^2+1 = +\infty \qquad \lim_{x \to \infty} x \cdot \ln(x) = +\infty$$

$$\lim_{x \to \infty} \frac{x^2+1}{x \cdot \ln(x)} = \lim_{x \to \infty} \frac{2x}{1+\ln(x)}$$

$$= \lim_{x \to \infty} \frac{2}{1/x}$$

$$= \lim_{x \to \infty} 2x$$

$$= +\infty$$

2. Manipulating Expressions so L'Hôpital's Rule will Apply.

- classic case:

$$\lim_{x \to a} f(x) \cdot g(x) = \lim_{x \to a} \frac{f(x)}{1/g(x)}$$

- Sometimes leads to an indeterminant form ($0/0$ or ∞/∞) that allows L'Hôpital's rule to be used.

Example

$$\lim_{x \to \infty} x^2 \cdot e^{-x} = \lim_{x \to \infty} \frac{x^2}{1/e^{-x}}$$

$$= \lim_{x \to \infty} \frac{x^2}{e^x} \quad \frac{\infty}{\infty}$$

$$= \lim_{x \to \infty} \frac{2x}{e^x} \quad \frac{\infty}{\infty}$$

$$= \lim_{x \to \infty} \frac{2}{e^x}$$

$$= 0.$$

Review Problem 3

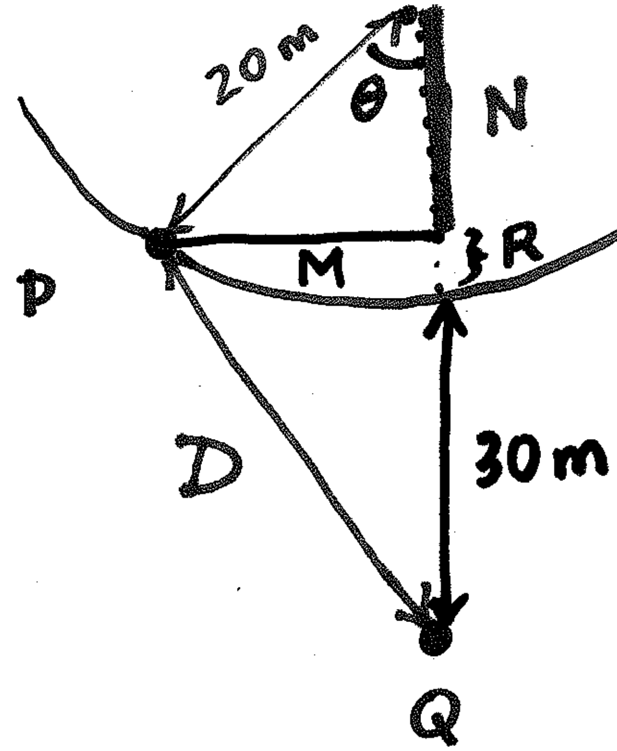
(a) One rotation every 5 minutes (300 seconds).

$$\text{Distance} = 2\pi r$$
$$= 2\pi(20)$$
$$= 40\pi$$
$$\approx 125.66 \text{ m.}$$

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}} = \frac{125.66}{300}$$
$$\approx 0.419 \text{ m/s.}$$

$$\text{Angular speed} = \frac{\text{Radians}}{\text{Time}}$$
$$= \frac{2\pi}{300}$$
$$= 0.021 \text{ radians/s.}$$

(b).



$\sin(\theta) = \dfrac{M}{20}$

or

$M = 20 \cdot \sin(\theta)$

$\cos(\theta) = \dfrac{N}{20}$

$N = 20 \cdot \cos(\theta)$

$R = 20 - N$
$= 20 - 20 \cdot \cos(\theta)$

Want: $\dfrac{dD}{dt}$

Have: $\dfrac{d\theta}{dt} \approx 0.021$ radians/s.

Set up: Relationship between D and θ.

Use Pythagoras:

$$D^2 = M^2 + (30 + R)^2$$

$$D^2 = (20 \cdot \sin(\theta))^2 + (30 + 20 - 20\cos(\theta))^2$$

$$D^2 = (20 \cdot \sin(\theta))^2 + (50 - 20\cos(\theta))^2$$

$$D = \sqrt{(20\cdot\sin(\theta))^2 + (50 - 20\cos(\theta))^2}$$

Chain Rule: $\quad \dfrac{dD}{dt} = \dfrac{dD}{d\theta}\cdot\dfrac{d\theta}{dt}$

$$\frac{dD}{d\theta} = \frac{2(20\sin(\theta))\cdot 20\cos(\theta) + 2(50-20\cos(\theta))\cdot 20\sin(\theta)}{2\sqrt{(20\sin(\theta))^2 + (50-20\cos(\theta))^2}}$$

(b) Plug $\theta = \pi/2$ into this and multiply by $d\theta/dt$.

$$\frac{dD}{dt} = \frac{2000}{2\sqrt{20^2+50^2}}\cdot(0.021) \approx 0.384 \text{ m/s}$$

(c) Plug $\theta = 0$ into this and multiply by $d\theta/dt$.

$$\frac{dD}{dt} = (0)(0.021) = 0 \text{ m/s}.$$

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$$\frac{dy}{dx} = \frac{-2x+4}{2y+7}$$

$$x^2+y^2-4x+7y=15.$$

c). $\frac{dy}{dx}=0$ $\quad -2x+4=0$

$$x = 2.$$

To find y:

$$(2)^2+y^2-4(2)+7y=15$$

$$y^2+7y-19=0$$

$$y = \frac{-7 \pm \sqrt{7^2-4(1)(-19)}}{2(1)}$$

$$\doteqdot -9.09 \;,\; 2.09.$$

Points with horizontal tangent:

$(2, -9.09)$ $\quad$ $(2, 2.09)$