Outline

1. Formulas for inverses.
2. Derivatives of inverse functions.
3. Solving equations involving or including logs.

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Next HW due Tuesday.

No class Thursday, Friday.

1. Finding Formulas for Inverses

- Start with: $y = f(x)$
- End with: $x = f^{-1}(y)$.

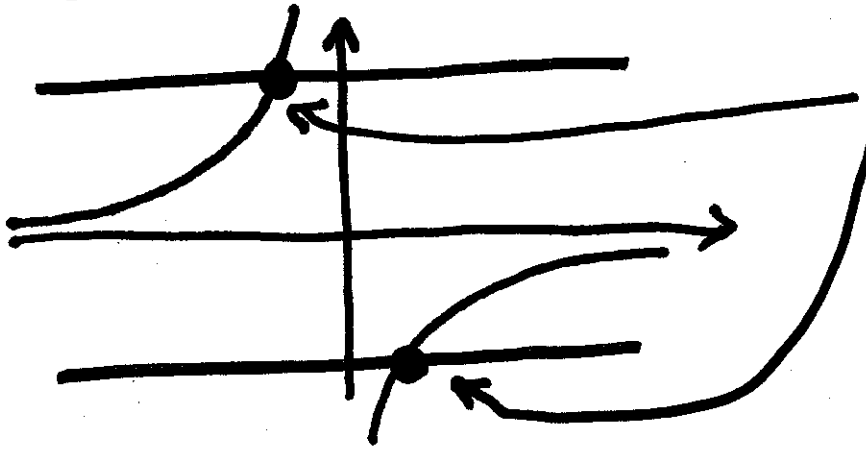
Example

Find a formula for the inverse of:

$$y = f(x) = \frac{1 + e^x}{1 - e^x}$$

Solution

① Use Horizontal Line Test to make sure a formula exists.



Each horizontal line cuts graph once only – $f(x)$ has an inverse.

② Write y for $f(x)$ in the formula. Then solve to make x the subject.

$$y = \frac{1 + e^x}{1 - e^x}$$

$$y \cdot (1 - e^x) = 1 + e^x$$

$$y - y \cdot e^x = 1 + e^x$$

$$y - 1 = e^x + y \cdot e^x$$

$$y - 1 = e^x \cdot (1 + y)$$

$$\frac{y-1}{1+y} = e^x$$

$$\ln\left(\frac{y-1}{1+y}\right) = \ln(e^x)$$

$$\ln\left(\frac{y-1}{1+y}\right) = x \cdot \underbrace{\ln(e)}_{=1}$$

$$x = \ln\left(\frac{y-1}{1+y}\right)$$

③ Replace x by $f^{-1}(y)$.

$$f^{-1}(y) = \ln\left(\frac{y-1}{1+y}\right).$$

2. Derivatives of Inverses

- You can find values of the derivative of the inverse, which is usually written: $(f^{-1})'(a)$.

$$(f^{-1})'(a) = \frac{1}{f'(\ f^{-1}(a)\ )}$$

↑ the derivative of the function

↑ plug in $f^{-1}(a)$.

Example

If: $f(x) = 3 + x^2 + \tan\left(\frac{\pi x}{2}\right)$

$a = 3.$

Find: $(f^{-1})'(a).$

Solution

① Evaluate $f^{-1}(a).$

Note: $f(0) = 3$ so $f^{-1}(3) = 0.$

② Find $f'(x).$

$$f'(x) = 0 + 2x + \sec^2\left(\frac{\pi x}{2}\right) \cdot \frac{\pi}{2}$$

③ Plug into: $(f^{-1})'(a) = \dfrac{1}{f'(f^{-1}(a))}$

$$(f^{-1})'(3) = \frac{1}{2(0) + \sec^2(0) \cdot \frac{\pi}{2}} = \frac{1}{\pi/2}$$

$$= 2/\pi$$

Example

Let x = # s'mores eaten per day

y = BMI of s'more eater.

Interpret:

(a) $f(3) = 32$

(b) $f^{-1}(40) = 7$

(c) $f'(3) = 3$

(d) $(f^{-1})'(45) = 1.$

Solution

(a) If you eat 3 s'mores you'll have a BMI of 32.

(b) If your BMI is 40, you eat 7 s'mores per day.

(c) If your s'more consumption goes up from 3 to 4 s'mores, your BMI will go up by approximately 3 points.

(d) If your BMI goes from 45 to 46, your daily s'more consumption will increase by approximately one s'more.

3. Equations that Involve Logarithms

$$\log(A^x) = x \cdot \log(A)$$

$$10^{\log(x)} = x$$

$$\log(A \cdot B) = \log(A) + \log(B)$$

$$\log\left(\frac{A}{B}\right) = \log(A) - \log(B).$$

Example

Solve: $\log(x) - \log(4-x) = 1.$

Solution

$$\log\left(\frac{x}{4-x}\right) = 1.$$

$$10^{\log\left(\frac{x}{4-x}\right)} = 10^{1}$$

$$\frac{x}{4-x} = 10$$

$$x = 10(4-x)$$

$$x = 40 - 10x$$

$$x + 10x = 40$$

$$11x = 40$$

$$x = 40/11.$$