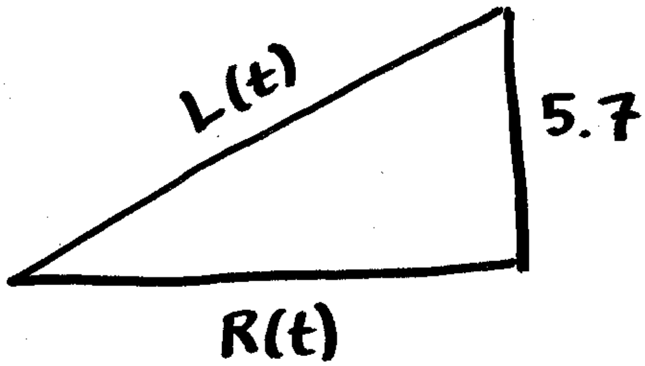
Outline

1. Finish related rates.
2. Exponential functions.
3. Finding formulas for exponential functions.

—‖—

Do-over: Wednesday
8-9 pm
9-10 pm
1212 Doherty.

1. Finish Related Rates



$$R(t) = \sqrt{L(t)^2 - 5.7^2}$$

$$R'(t) = \frac{L(t)}{R(t)} \cdot L'(t).$$

Have: $L(t) = 10$ miles.

$L'(t) = -420$ miles/hour

$$R(t) = \sqrt{10^2 - 5.7^2} = 8.2 \text{ miles.}$$

$$R'(t) = \frac{10}{8.2}(-420) = -511 \text{ miles/hour.}$$

2. Exponential Functions

- Functions whose formula fits the pattern:

$$y = A \cdot B^{x}$$

A: a number, usually called the initial value

B: a number, usually called the growth factor.

x: variable is in exponent.

- To calculate the growth factor, B, need two points: (x_1, y_1) and (x_2, y_2).

$$B = \left(\frac{y_2}{y_1}\right)^{\frac{1}{x_2 - x_1}}$$

- If you have a table of values, and you suspect that an exponential function would be a good function to connect x and y, you can test this by:

1. Calculate $B = \left(\frac{y_2}{y_1}\right)^{\frac{1}{x_2 - x_1}}$

   for several different pairs of points.

2. If you always get the same B-value, an exponential function will do a good job of connecting x and y.

Handout 11: Exponential Functions

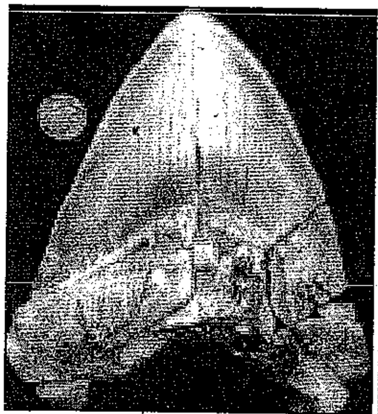


Figure 1: Fossilized *C. megalodon* tooth. (The small circle is a dime.)

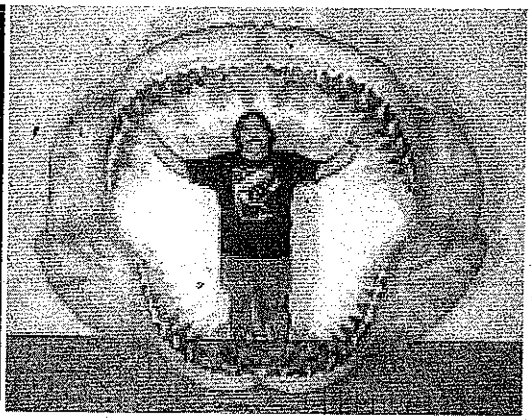


Figure 2: Replica of a *C. megalodon* jaw. (The replica is about seven and a half feet tall.)

The "mega-tooth" shark (*Carcharodon megalodon*) is a giant shark that appears to have lived between 10 and 50 million years ago[1]. Much of what we know about this shark comes from fossilized teeth (see Figures 1 and 2[2]) that have been found in coastal regions of Virginia, North Carolina, South Carolina, Georgia and Florida. Based on the size of these teeth, many scientists believe that *C. megalodon* was approximately the size of a Greyhound bus[3] (see Figures 4(a)[4] and 4(b)[5]).

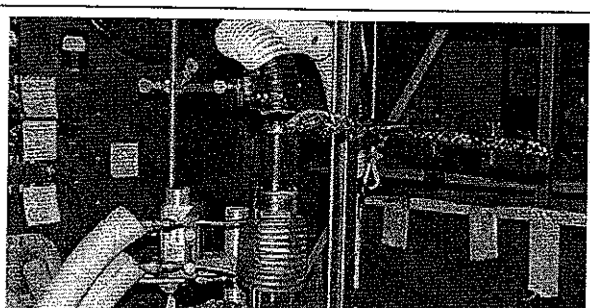


Figure 3: Part of the apparatus used for potassium-argon dating at the Earth Sciences Laboratory at Dartmouth College. This particular item is an induction furnace which is used to extract potassium-40 from the sediment sample.

Many scientists believe that *C. megalodon* died out a long time ago. This view is supported by radioactive dating of the sediments that *C. megalodon* teeth are found in. The specific technique employed for megalodon teeth is called potassium-argon dating.

In this technique, a sample of rock is chemically analyzed and the percentage of the rock that is potassium-40 determined (see Figure 3[6]). Typically for fresh sediment, 2.4% of the sediment is potassium-40. Potassium-40 is a radioactive isotope (half life = 1260 million years) of potassium that decays into the inert gas argon-40. This decay process occurs at a constant percentage rate of change.

---

[1] Ellis, R. and McCosker, J. Eds. 1995. *Great White Shark*. Palo Alto, CA: Stanford University Press.
[2] Image source for Figures 1 and 2: http://sharksteeth.com
[3] Gottfried, M.D., L.J.V. Campagno & S.C. Bowman. 1996. Size and skeletal anatomy of the giant "Megatooth" shark *Carcharodon megalodon*. In A.P. Klimley and D.G. Ainley. Eds. *Great White Sharks: The Biology of* Carcharodon carcharias. San Diego, CA: Academic Press.
[4] Image source: http://www.csuohio.edu/interual/new_stu_transport.html
[5] Image source: http://hometown.aol.com/rjrasalli/index5.html
[6] Image source: http://www.darmouth.edu/~earthsci/labs/KArlab.htm

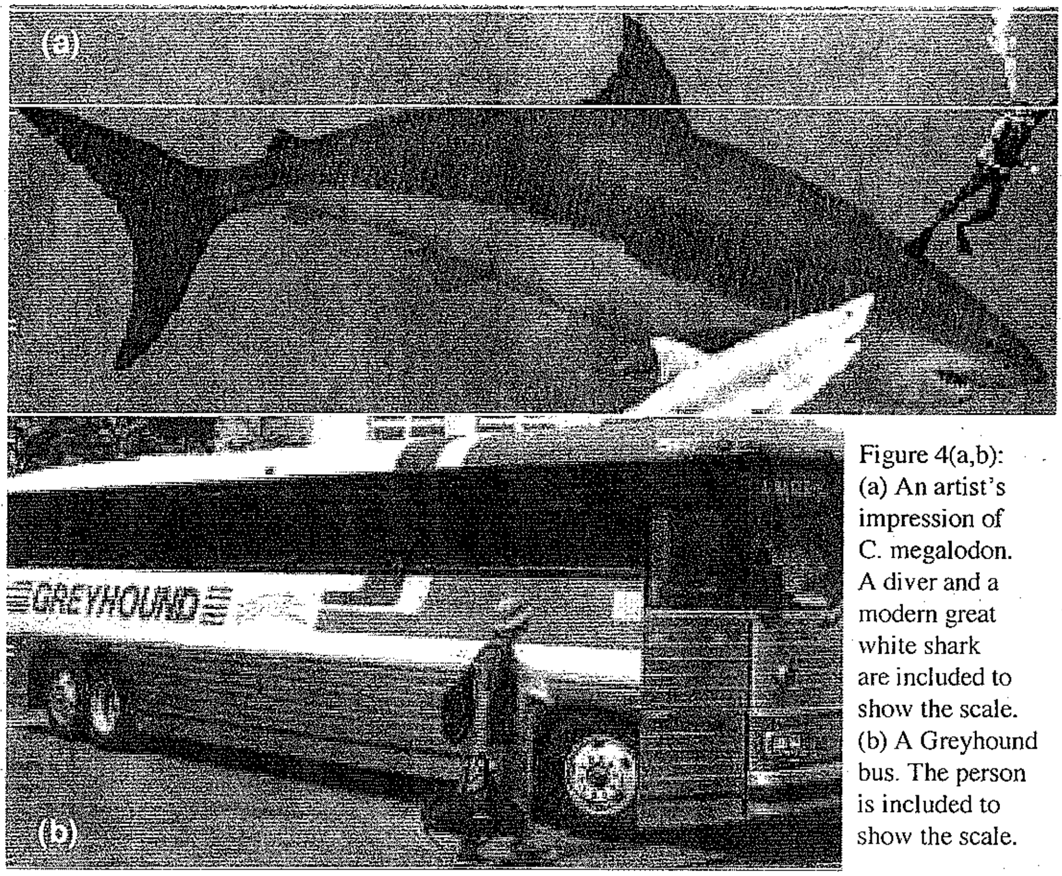


Figure 4(a,b): (a) An artist's impression of C. megalodon. A diver and a modern great white shark are included to show the scale. (b) A Greyhound bus. The person is included to show the scale.

(a) Use the information given on potassium-40 and its decay to complete Table 1 (below).

Age of sediment (millions of years)	0	1260	2520
Percentage of Potassium-40	2.4	1.2	0.6

Table 1.

(b) Is the data recorded in Table 2 perfectly exponential or not?

(0, 2.4) and (1260, 1.2) B = 0.999450034 5

(1260, 1.2) and (2520, 0.6) B = 0.999450034 5

(0, 2.4) and (2520, 0.6) B = 0.999450034 5

(c) What sort of function will do a good job of representing this situation? Be careful to explain your reasoning.

Exponential function $y = A \cdot B^x$.

(d) Find a formula for the function that gives the percentage of potassium-40 as a function of the age of the rock sample (expressed in millions of years).

Let $y =$ % of potassium-40.

$x =$ age (millions of years).

$y = A \cdot (0.9994500345)^x$. To get A plug in $x = 1260$ $y = 1.2$

$1.2 = A(0.9994500345)^{1260}$ $A = \dfrac{1.2}{(0.9994500345)^{1260}} = 2.4$

$\boxed{y = (2.4)\cdot(0.9994500345)^x}$

(e) A *C. megalodon* tooth was found in a quarry in North Carolina. Lab tests determined that 2.38683% of the sediment that the tooth was found was potassium-40. Set up an equation that would give you the age of the tooth if you were to solve the equation.

$2.38683 = (2.4)(0.9994500345)^x$ Solve for x.

Using calculator:

$x = 10.002624$

(f) Use your graphing calculator to find the approximate solution of the equation that you set up in Part (e). Use the viewing window:

Xmin = 0 Xmax = 20 Ymin = 2.3 Ymax = 2.5

The age of the tooth is slightly over 10 million years.