Outline

1. Overview of related rates.
2. Jack the Ripper.
3. Gulf of Sidra.

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Do-over test: Wednesday
8-9 pm
9-10 pm
1212 Doherty.

1. Structure of Related Rates Problems

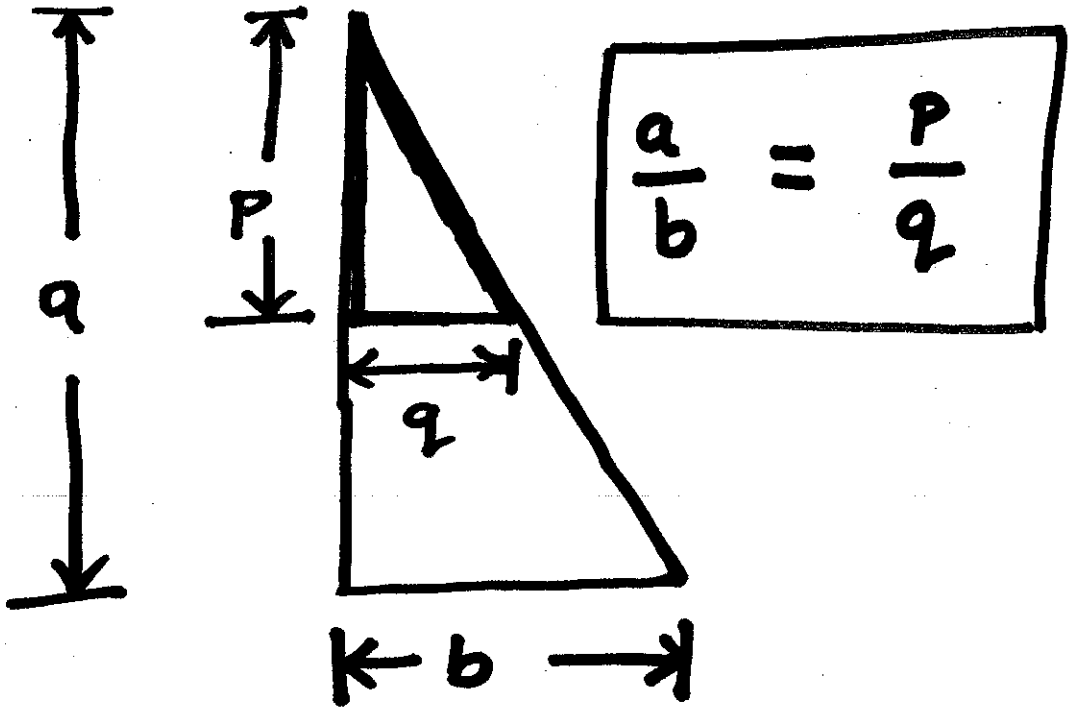
- Overall strategy:

(a) Figure out what rate we need to find. (Say dy/dt).

(b) Figure out what rate we are given (say dx/dt).

(c) Find an equation that connects the quantities $x(t)$ and $y(t)$.

- Volume and area formulas.
- Trigonometric formulas.
- Pythagoras $a^2 + b^2 = c^2$
- Similar triangles.

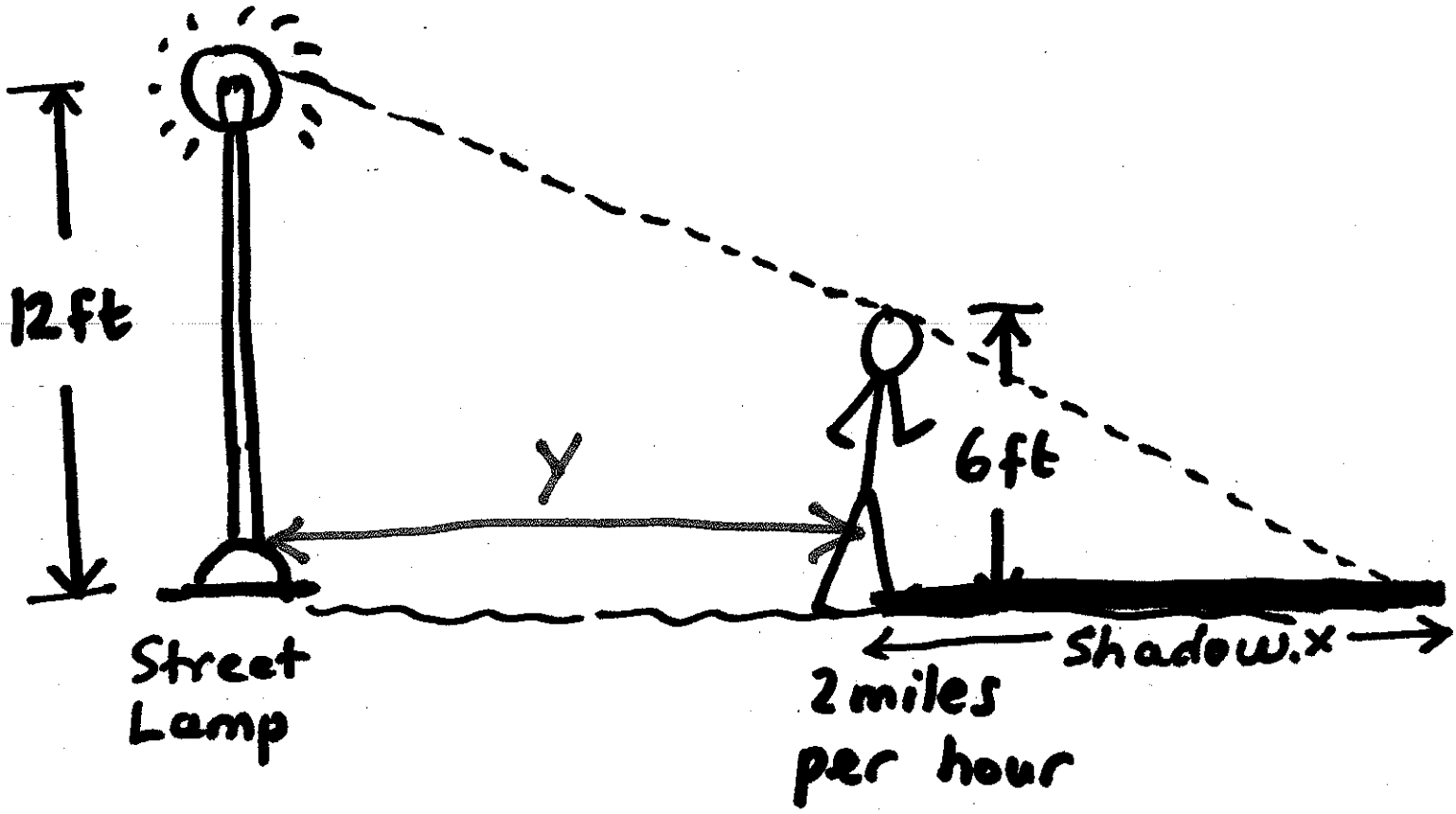


$$\frac{a}{b} = \frac{p}{q}$$

(d) Differentiate the equation so that it becomes an equation relating $\frac{dx}{dt}$ and $\frac{dy}{dt}$.

(e) Solve for the derivate / rate you need, i.e. dy/dt.

Example



$$\frac{10560}{3600} = 2.93 \text{ ft/s.}$$

When he is 30 ft from the lamp, how quickly is his shadow growing?

Solution

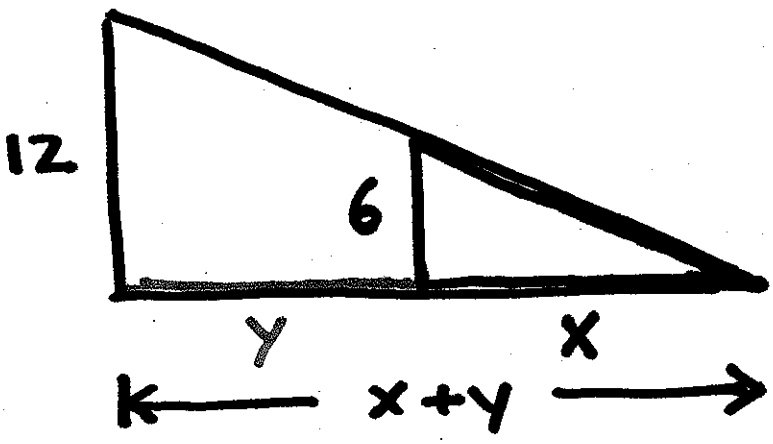
We want the rate of change of the length of the shadow.

x = length of shadow (feet).

Want: dx/dt.

y = distance from lamp to person (feet).

Have: $dy/dt = 2.93$ ft/s.



Similar Triangles:

$$\frac{x+y}{12} = \frac{x}{6}$$

$$x + y = 2x$$

$$y = x$$

$$y(t) = x(t).$$

Take derivatives:

$$y'(t) = x'(t)$$

$$\frac{dy}{dt} = \frac{dx}{dt}$$

} only one of these necessary

Want: $\frac{dx}{dt}$

Have: $\frac{dy}{dt} = 2.93 \text{ ft/s}$

Answer: $\frac{dx}{dt} = 2.93 \text{ ft/s}.$

Handout 10: The Long Shadow of Jack the Ripper

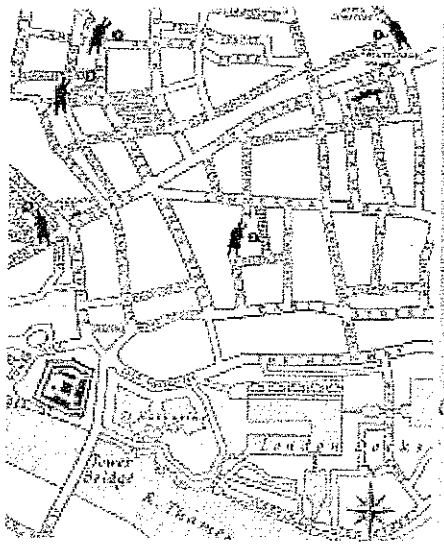


Figure 1: Locations at which the five victims generally attributed to Jack the Ripper were found.

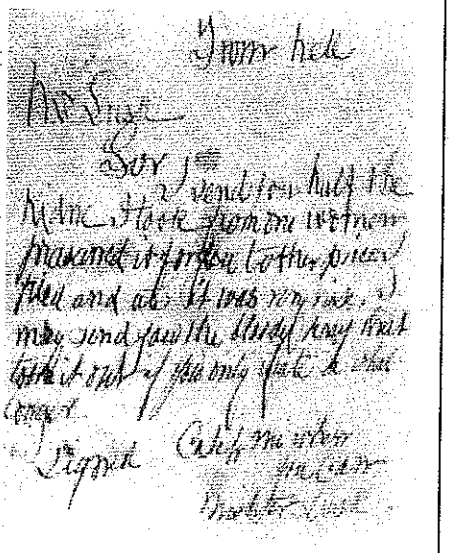


Figure 2: Note sent to police at the time of the Ripper murders. Experts believe that the note was probably *not* written by the killer.

Perhaps the most famous series of unsolved murders in history are the crimes of Jack the Ripper. "Jack the Ripper" is the nickname that has been given to a serial killer who murdered several prostitutes in the East End of London in 1888 (see Figure 1[1]). The nickname of "Jack" originated from a note (see Figure 2) that was written by someone claiming to be the killer, although the note has never been definitely authenticated. At the time of the slayings, the killer was also known as "The Whitechapel Murderer" and "Leather Apron."

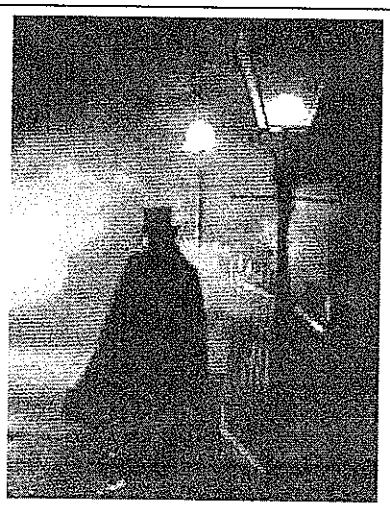


Figure 3: A menacing figure appears out of the mist, a long shadow cast by the street lamps.

There is considerable debate over the exact number of prostitutes that Jack the Ripper actually killed. Many authorities believe that one killer was responsible for at least five slayings, although some attribute as many as fourteen deaths to the ripper. The five victims of these brutal murders were:

1. Mary Ann (Polly) Nichols, murdered Friday August 31, 1888.
2. Annie Chapman, murdered Saturday September 8, 1888.
3. Elizabeth Stride, murdered Sunday September 30, 1888.
4. Catherine Eddowes, also murdered Sunday September 20, 1888.
5. Mary Jane Kelly, murdered Friday November 9, 1888. (Played by Heather Graham in the recent movie, "From Hell.")

Perhaps the signature scene of the many "Jack the Ripper" movies and television programs[2] consists of a young woman walking the dark and misty streets of London, her way illuminated by a single street lamp. Suddenly, a male figure in a cape and top hat coalesces from out of the fog. His shadow stretches out, long and ominous before him (see Figure 3[3]). Perhaps sensing danger, the young woman turns. The shadow

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[1] Image source: http://www.travelbritain.com/

[2] Legendary British rock group "Spinal Tap" even proposed a rock opera based on the ripper murders. Although the project was never completed, the working title was "Saucy Jack."

[3] Image source: http://www.timbosliverpool.co.uk/

looms, almost gripping her with sinister, unholy power. Her pretty face contorts in a paroxysm of fear, her lips tremble, too afraid to scream. He is upon her. Darkness descends, and the caped figure dissolves into the mist, phantasmal, leaving death and depravity in its evil wake.

One of the effects that contributes to the atmosphere, suspense and a palpable sense of menace is the way that the murderer's shadow seems to move forward at a much faster rate than the person is actually walking. In this handout, you will use derivatives and the Principle of Similar Triangles to determine why this happens.

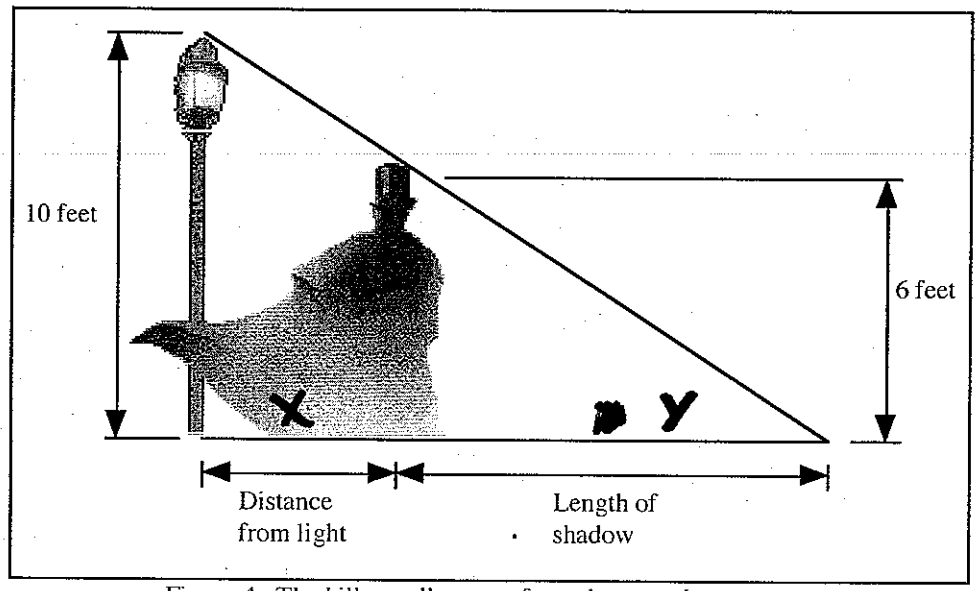

Figure 4: The killer walks away from the street lamp.

1. Figure 4 shows the situation as the killer walks away from the street lamp. The lamp is 18 feet tall, and the killer (including top hat) is 6 feet tall. If x represents the killer's distance from the street lamp in feet and y represents the length of the killer's shadow in feet, find a relationship between x and y.

$$\frac{x+y}{10} = \frac{y}{6}$$

$$6x + 6y = 10y$$

$$6x = 4y$$

$$x = \frac{2}{3}y$$

$$y = \frac{3}{2}x$$

2. A normal human walking speed is about 4 feet per second. If this is how fast the killer is walking, how would you represent this fact using the symbols defined in Question 1 and derivatives?

$$dx/dt = 4 \text{ ft/s.}$$

3. The ultimate objective of this problem is to find out how quickly the length of the killer's shadow grows. What quantity or derivative do you need to calculate?

$$dy/dt$$

4. Find a relationship between the derivatives $\frac{dx}{dt}$ and $\frac{dy}{dt}$.

$$\frac{dy}{dt} = \frac{3}{2}\frac{dx}{dt}$$

5. How quickly (in units of feet per second) is the killer's shadow growing when he is ten feet from the street lamp?

$$\frac{dy}{dt} = 6 \text{ ft/s.}$$

Handout 9: The Gulf of Sidra Incident

Figure 1: Libyan President Colonel Muammar Abu Minyar al-QADHAFI, better known to the west as Gadaffi.

In 1981 Colonel Gadaffi (the President of Libya - see Figure 1[1]) claimed the entire Gulf of Sidra as Libyan territorial waters (see Figure 2[2]).

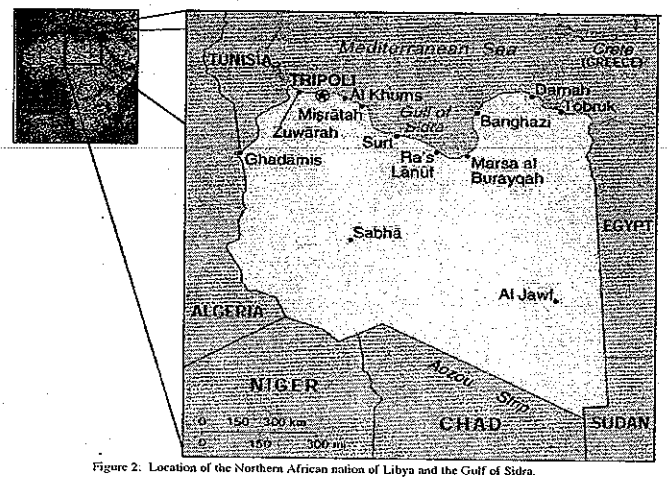

Figure 2: Location of the Northern African nation of Libya and the Gulf of Sidra.

This action was not in accord with international laws, and President Reagan (see Figure 3[3]) challenged this position by conducting naval exercises in the Gulf[4].

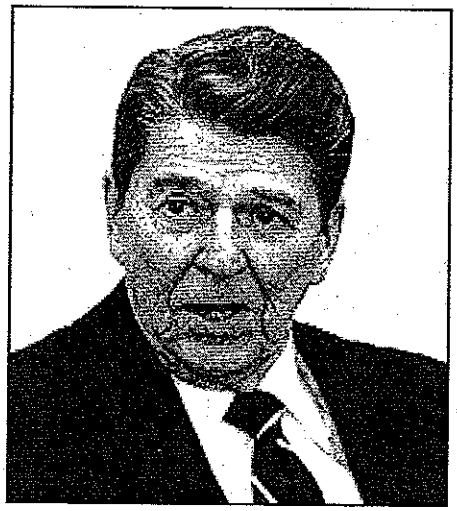

Figure 3: Former US President Ronald Reagan.

The admiral commanding these naval forces ordered F-14's (see Figure 4[5]) to patrol at a distance of ten miles from the Libyan coast. Any unidentified aircraft crossing the Libyan coast at a speed of greater than 500 miles per hour was to be treated as hostile.

Figure 5 (see next page) shows one of the situations that developed during this crisis. An F-14 was circling (essentially remaining near the same spot) at an altitude of approximately 5.7 miles. When the F-14 was 10 miles (diagonal distance) from the coast, its AWG-9 air search radar detected an object moving rapidly across the Libyan coastline.

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[1] Image source: http://www.goal.com/es/esp/

[2] Image sources: CIA World Factbook, 2002. and http://www.africaguide.com/

[3] Image source: http://reagan.webteamone.com/reagan_images.cfm

[4] Throughout these exercises, Libyan and US forces antagonized each other by approaching with every indication that they were preparing to attack. The most serious incident occurred on August 19, 1981, when a pair of Libyan Su-22 "Fitter" aircraft approached a pair of US Navy F-14's and fired an air-to-air missile. Both F-14's easily evaded the missile and returned fire destroying both Libyan aircraft.

[5] Image source: US Navy, http://www.navy.mil/

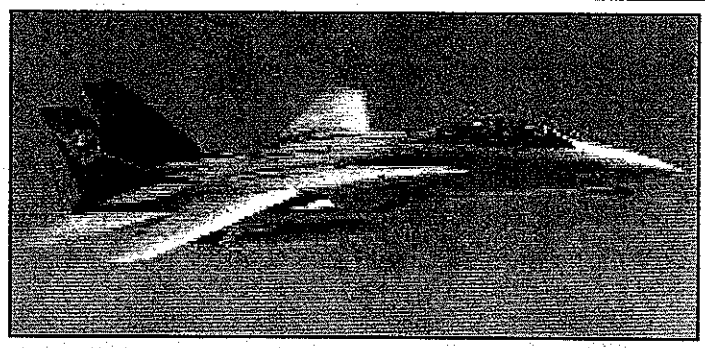

Figure 4: The F-14 Tomcat Fleet Defense Interceptor

The AWG-9 measures the rate at which the distance between the F-14 and the target changes. In this situation, the AWG-9 measured the rate of change of this distance to be 420 miles per hour.

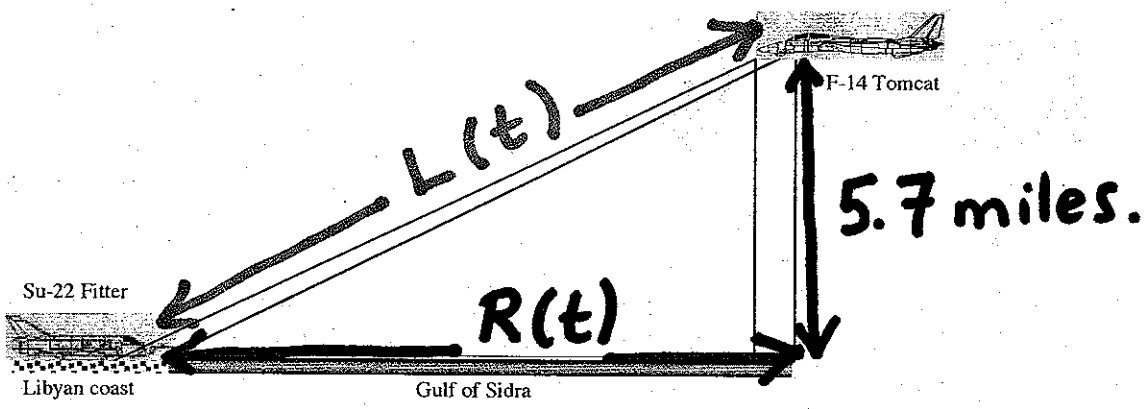

Figure 5: The situation at the start of the engagement.

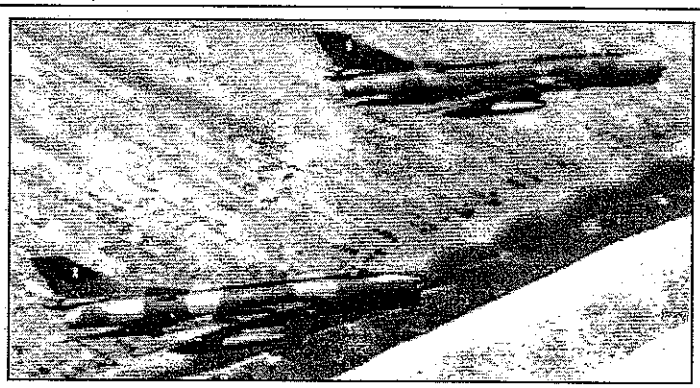

Figure 6: A pair of Libyan Su-22 "Fitter" aircraft flying over a coastal region near the Gulf of Sidra.

In 1981, the Libyan airforce included a number of high performance Soviet made aircraft such as the Su-22 "Fitter" (see Figure 6[6]). The top speed[7] of the Su-22 was approximately 1380 miles per hour.

[6] Image source: http://www.tbe.aims.on.ca/keele/emir/su-22.htm
[7] Source: http://www.acrospaceweb.org/

In this handout, you will analyze the mathematics that the computer on the F-14 performed to covert the information from the AWG-9 radar into information about the speed of the object that the F-14 detected. When you have finished doing this, you will be able to decide whether the radar contact should be classified as "hostile" or not.

Let t represent the number of seconds that have elapsed since the F-14 first detected the target. Let $L(t)$ represent the diagonal distance (the length of the hypotenuse of the triangle in Figure 5) between the F-14 and the object. Let $R(t)$ represent the horizontal distance between the F-14 and the object (i.e. the length of the base of the triangle in Figure 5).

1. Using the quantities defined above to express your answer, what rate of change does the AWG-9 radar measure?

$L'(t)$ or $\frac{dL}{dt}$.

2. Using the quantities defined above to express your answer, what is the air speed of the object that the F-14 detected?

$R'(t)$ or $\frac{dR}{dt}$.

3. Using Figure 5, find a relationship between the functions $R(t)$ and $L(t)$. Use this relationship to find a relationship between the derivatives $R'(t)$ and $L'(t)$.

Pythagorean Theorem.

$$(R(t))^2 + (5.7)^2 = (L(t))^2$$

$$2 \cdot R(t) \cdot R'(t) + 0 = 2 \cdot L(t) \cdot L'(t)$$

$$R'(t) = \frac{L(t)}{R(t)} \cdot L'(t)$$

$$R(t) = \sqrt{L(t)^2 - 5.7^2}$$

4. This incident occurred when the diagonal distance between the F-14 and the Libyan cost was 10 miles. What was the horizontal distance between the F-14 and the object at the moment when the object crossed the Libyan coastline?

$$R(t) = \sqrt{10^2 - 5.7^2} \approx 8.216 \text{ miles}$$

5. Use your answers to Questions 1-4, along with any other relevant information from the introduction, to calculate the air speed of the object as it crossed the Libyan coastline. In the actual incident, the F-14 crew classified the contact as hostile. Was their decision justified?

$$R'(t) = \frac{10}{8.216}(420) \approx 511.17 \text{ miles/hour}$$

The decision was justified.