Outline

1. The Chain Rule and Lollipops.
2. Related rates problems.
3. Feedback forms.

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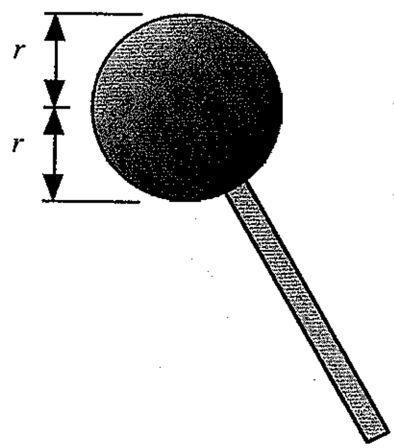
Remember: Gateway attempt #2 in recitation Thursday.

Handout 8: An Experimental Test of the Chain Rule

Figure 1 shows a (roughly) spherical lollipop. The relationship between the volume, V, and the radius, r, is also given.



The volume, V, of a sphere with radius r is given by:

$$V = \frac{4}{3} \cdot \pi \cdot r^3$$

The volume and the radius of the sphere change with time, t, so that volume and radius are functions of time, $V(t)$ and $r(t)$, then the relationship is given by:

$$V(t) = \frac{4}{3} \cdot \pi \cdot \left[r(t)\right]^3$$

Figure 1: Lollipop and volume formulas.

1. If you were to eat an lollipop like the one shown in Figure 1, you would expect both the radius and the volume of the lollipop to decrease the longer that you sucked. In this situation, the volume could be thought of as a function of time, $V(t)$, and the radius could also be thought of as a function of time, $r(t)$. Find a relationship between the functions $V(t)$ and $r(t)$.

$$V(t) = \frac{4}{3}\pi\left(r(t)\right)^3$$

2. Based on the relationship that you have found, how would the derivatives $V'(t)$ and $r'(t)$ be related?

$$V'(t) = \frac{4}{3}\pi \cdot 3\left(r(t)\right)^2 \cdot r'(t)$$

3. Now, you are going to put this theoretical prediction to the test. With the help of a few of your classmates, measure the radius of your lollipop at various times and record the results in the table provided below.

Time sucked (seconds)	Radius of lollipop (mm)	Rate of change of radius (mm/second)	$4\pi r(t)^2 \cdot r'(t)$
0	20	$\frac{-2}{20}$	[redacted] -502.65
20	18	$\frac{-3}{20}$	-407.15
40	15	$\frac{-1}{20}$	-141.37
60	14	$\frac{0}{20}$	Too small to measure.
80	14		

4. Use the results that you have recorded to fill in the table given below.

Time sucked (seconds)	Volume of lollipop (cubic mm)	Rate of change of volume (cubic mm/second)
0	33,510	-454.05
20	24,429	-514.6
40	14,137	-132.15
60	11,494	

5. Compare the right columns of the two tables. Do the numbers in these columns support the theoretical prediction? Explain why or why not.

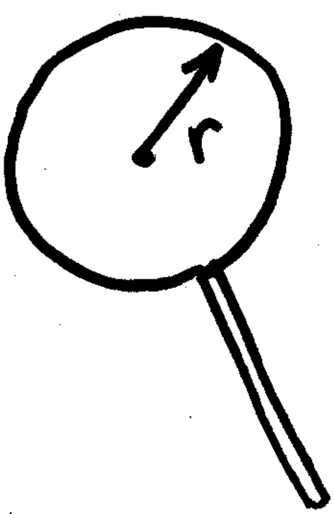
1. Chain Rule and Lollipops

- Data we collected suggests some kind of relationship between the quantities:

  $V'(t)$ = rate of change of volume.

  $4 \cdot \pi \cdot r(t)^2 \cdot r'(t)$ = concoction involving rate of change of radius.

- Question: Why is there a connection?



$r(t)$ = radius after t seconds of sucking.

$V(t)$ = volume after t seconds of sucking.

For a sphere: $V = \frac{4}{3}\pi r^3$

Put $V(t)$ and $r(t)$ into this:

$$V(t) = \frac{4}{3}\pi\left(r(t)\right)^3$$

- To get an equation connecting the derivatives $V'(t)$ and $r'(t)$, take derivatives of both sides.

$$V'(t) = \frac{4}{3}\pi \cdot 3\left(r(t)\right)^2 \cdot r'(t)$$

$$V'(t) = 4\pi r(t)^2 \cdot r'(t).$$

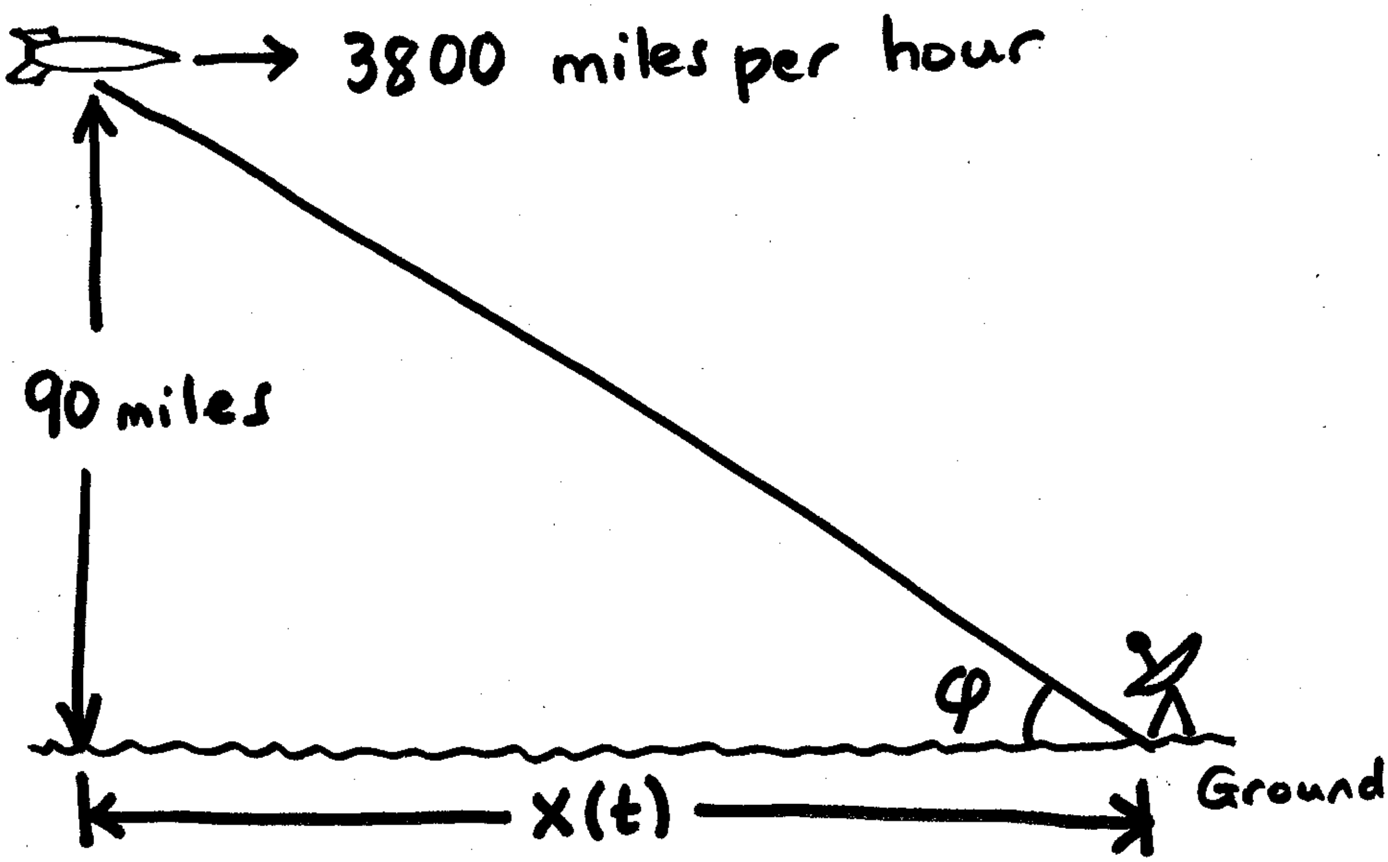
- Related rates situation. An equation that tells us how to calculate one derivative (say $V'(t)$) from another derivative ($r'(t)$).

Example

- A radar dish tracks an incoming missile. The missile flies at an altitude of 90 miles and with a speed of 3800 miles per hour.
- How quickly does the radar have to turn to stay locked on to the missile:

  (a) If detected 200 miles away

  (b) If missile is directly overhead?



Question we must answer:

- What is the value of $\frac{d\varphi}{dt}$?

<u>Solution:</u> $$\frac{90}{x(t)} = \tan(\varphi(t))$$

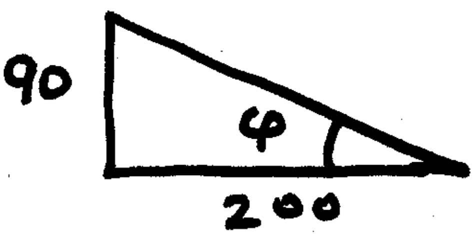
φ changes with time.

- Take derivative of both sides:

$$\frac{(0)\cdot x(t) - (90)\cdot x'(t)}{(x(t))^2} = \frac{1}{\cos^2(\varphi(t))}\cdot \varphi'(t)$$

$$\varphi'(t) = \frac{d\varphi}{dt} = \frac{-90 \cdot x'(t) \cdot \cos^2(\varphi(t))}{(x(t))^2}$$

(a) Missile detected at range of 200 miles.



$$x'(t) = -3800$$

$$x(t) = 200$$

$$\varphi(t) = \tan^{-1}\left(\frac{90}{200}\right)$$

$$= 24.2^\circ$$

so $\varphi'(t)$ will be:

$$\varphi'(t) = \frac{(-90)(-3800) \cdot \cos^2(24.2^\circ)}{(200)^2}$$

$$= 7.11 \ ^\circ \text{ per hour.}$$