Outline

1. New derivative rules.

2. Implicit differentiation.

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- HW #4 due in recitation.
- Gateways start this week.
- Do-over: Wednesday 2/25

  8 - 9 pm
  9 - 10 pm 1212 DH

1. New differentiation Rules

Exponential Function

$$f(x) = e^x$$

Derivatives

$$f'(x) = e^x$$

$e \approx 2.718\ldots$

Logarithm (natural) Function

$$f(x) = \ln(x)$$

Derivatives

$$f'(x) = \frac{1}{(x)}$$

Examples

Find $f'(x)$ when:

(a) $f(x) = 7 \cdot e^{x^2 - 3x + 2}$

(b) $f(x) = \ln\left(1 + \cos^2(x)\right).$

Solution:

(a) $f(x) = 7 \cdot e^{(x^2 - 3x + 2)}$

$$f'(x) = 7 \cdot e^{(x^2-3x+2)} \cdot (2x-3)$$

(b) $f(x) = \ln\left(1 + (\cos(x))^2\right)$

$$f'(x) = \frac{1}{\left(1 + (\cos(x))^2\right)} \cdot \left(0 + 2(\cos(x))^{1}(-\sin(x))\right)$$

2. Implicit Differentiation

- Useful when finding dy/dx for an equation where the x's and y's are mixed together and can't be separated:

all y's = all x's

e.g. $x^2 + x \cdot y + y^2 = 10.$

Example + Procedure

Find $\dfrac{dy}{dx}$ for :

$$x^2 + x \cdot y + y^2 = 10.$$

Solution

① Replace each y by $f(x)$ in the equation.

$$x^2 + x \cdot f(x) + (f(x))^2 = 10.$$

② Take derivative of each term in the equation with respect to x.

$$2x + 1 \cdot f(x) + x \cdot f'(x) + 2 \cdot f(x) \cdot f'(x) = 0$$

$\Rightarrow$

$\rightarrow$

$\rightarrow$

③ Move every term with $f'(x)$ in it to one side of the equation ; every term that doesn't have $f'(x)$ gets moved to the other side of the equation.

$$x \cdot f'(x) + 2 \cdot f(x) \cdot f'(x) = -2x - f(x)$$

④ Factor out $f'(x)$ and make $f'(x)$ the subject of the equation.

$$f'(x) \cdot \left[x + 2 \cdot f(x) \right] = -2x - f(x)$$

$$f'(x) = \frac{-2x - f(x)}{x + 2 \cdot f(x)}$$

⑤ Replace $f'(x)$ by dy/dx and $f(x)$ by y.

$$\frac{dy}{dx} = \frac{-2x - y}{x + 2 \cdot y}$$

Example

Find $\frac{dy}{dx}$ for: $\sin(x \cdot y) + x^2 - y^3 = \frac{x}{y}$.

Solution

① $\sin(x \cdot f(x)) + x^2 - (f(x))^3 = \frac{x}{f(x)}$

② $\cos(x \cdot f(x))\left(x \cdot f'(x) + f(x)\right) + 2x +$

$-3(f(x))^2 \cdot f'(x)$

$$= \frac{f(x) \cdot 1 - x \cdot f'(x)}{f(x)^2}$$

③ $x \cdot f'(x) \cdot \cos(x \cdot f(x)) + f(x) \cdot \cos(x \cdot f(x))$

$+ 2x - 3f(x)^2 \cdot f'(x) = \frac{f(x)}{f(x)^2} - \frac{x \cdot f'(x)}{f(x)^2}$

$$x \cdot f'(x) \cdot \cos\big(x \cdot f(x)\big) - 3\, f(x)^2 \cdot f'(x) + \frac{x \cdot f'(x)}{f(x)^2}$$

$$= \frac{1}{f(x)} - f(x) \cdot \cos\big(x \cdot f(x)\big) - 2x$$

$f(x) \neq 0$

$$④\; f'(x) \cdot \left[x \cdot \cos\big(x \cdot f(x)\big) - 3 f(x)^2 + \frac{x}{f(x)^2} \right]$$

$$= \frac{1}{f(x)} - f(x) \cdot \cos\big(x \cdot f(x)\big) - 2x$$

$$f'(x) = \frac{\dfrac{1}{f(x)} - f(x) \cdot \cos\big(x \cdot f(x)\big) - 2x}{x \cdot \cos\big(x \cdot f(x)\big) - 3 f(x)^2 + \dfrac{x}{f(x)^2}}$$

⑤ $$\frac{dy}{dx} = \frac{\frac{1}{y} - y \cdot \cos(xy) - 2x}{x \cdot \cos(xy) - 3y^2 + \frac{x}{y^2}}$$