

Outline

1. New derivative rules.
2. Implicit differentiation.

— II —

- HW #4 due in recitation.
- Gateways start this week.
- Do-over: Wednesday 2/25
8-9 pm 1212 DH
9-10 pm

I. New differentiation Rules

Exponential Function

$$f(x) = e^x$$

Derivatives

$$f'(x) = e^x$$

$$e \approx 2.718\dots$$

Examples

Find $f'(x)$ when:

$$(a) f(x) = 7 \cdot e^{x^2 - 3x + 2}$$

$$(b) f(x) = \ln(1 + \cos^2(x)).$$

Logarithm (natural) Function

$$f(x) = \ln(x)$$

Derivatives

$$f'(x) = \frac{1}{x}$$

Solution:

$$(x^2 - 3x + 2)$$

(a) $f(x) = 7 \cdot e^{(x^2 - 3x + 2)}$

$$f'(x) = 7 \cdot e^{(x^2 - 3x + 2)} \cdot (2x - 3)$$

(b) $f(x) = \ln(1 + (\cos(x))^2)$

$$f'(x) = \frac{1}{(1 + (\cos(x))^2)} \cdot (0 + 2(\cos(x))(-\sin(x)))$$

2. Implicit Differentiation

- Useful when finding $\frac{dy}{dx}$ for an equation where the x 's and y 's are mixed together and can't be separated:
$$\text{all } y's = \text{all } x's$$

e.g. $x^2 + x \cdot y + y^2 = 10.$

Example + Procedure

Find $\frac{dy}{dx}$ for :

$$x^2 + x \cdot y + y^2 = 10.$$

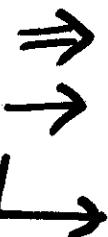
Solution

① Replace each y by $f(x)$ in the equation.

$$x^2 + x \cdot f(x) + (f(x))^2 = 10.$$

② Take derivative of each term in the equation with respect to x .

$$2x + 1 \cdot f(x) + x \cdot f'(x) + 2 \cdot f(x) \cdot f'(x) = 0$$



③ Move every term with $f'(x)$ in it to one side of the equation ; every term that doesn't have $f'(x)$ gets moved to the other side of the equation.

$$x \cdot f'(x) + 2 \cdot f(x) \cdot f'(x) = -2x - f(x)$$

④ Factor out $f'(x)$ and make $f'(x)$ the subject of the equation.

$$f'(x) \cdot [x + 2 \cdot f(x)] = -2x - f(x)$$

$$f'(x) = \frac{-2x - f(x)}{x + 2 \cdot f(x)}$$

⑤ Replace $f'(x)$ by $\frac{dy}{dx}$ and $f(x)$ by y .

$$\frac{dy}{dx} = \frac{-2x - y}{x + 2y}$$

Example

Find $\frac{dy}{dx}$ for: $\sin(x \cdot y) + x^2 - y^3 = \frac{x}{y}.$

Solution

$$\textcircled{1} \quad \sin(x \cdot f(x)) + x^2 - (f(x))^3 = \frac{x}{f(x)}$$

$$\textcircled{2} \quad \cos(x \cdot f(x)) (x \cdot f'(x) + f(x)) + 2x +$$

$$- 3(f(x))^2 \cdot f'(x)$$

$$= \frac{f(x) \cdot 1 - x \cdot f'(x)}{f(x)^2}$$

$$\textcircled{3} \quad x \cdot f'(x) \cdot \cos(x \cdot f(x)) + f(x) \cdot \cos(x \cdot f(x))$$

$$+ 2x - 3f(x)^2 \cdot f'(x) = \frac{f(x)}{f(x)^2} - \frac{x \cdot f'(x)}{f(x)^2}$$

$$x \cdot f'(x) \cdot \cos(x \cdot f(x)) - 3 f(x)^2 \cdot f'(x) + \frac{x \cdot f'(x)}{f(x)^2}$$

$$= \frac{1}{f(x)} - f(x) \cdot \cos(x \cdot f(x)) - 2x$$

$$f(x) \neq 0$$

$$\textcircled{4} \quad f'(x) \cdot \left[x \cdot \cos(x \cdot f(x)) - 3 f(x)^2 + \frac{x}{f(x)^2} \right]$$

$$= \frac{1}{f(x)} - f(x) \cdot \cos(x \cdot f(x)) - 2x$$

$$f'(x) = \frac{\frac{1}{f(x)} - f(x) \cdot \cos(x \cdot f(x)) - 2x}{x \cdot \cos(x \cdot f(x)) - 3 f(x)^2 + \frac{x}{f(x)^2}}$$

(5) $\frac{dy}{dx} = \frac{\frac{1}{y} - y \cdot \cos(xy) - 2x}{x \cdot \cos(xy) - 3y^2 + \frac{x}{y^2}}$