Outline

1. Product & quotient rules.
2. dy/dx notation.
3. The Chain Rule.

No HW, no quiz this week.

Office hours Tuesday: 11am – 1pm.

1. Product & Quotient Rules

$$\big(f(x)\cdot g(x)\big)' = f'(x)\cdot g(x) + f(x)\cdot g'(x)$$

$$\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)\cdot g(x) - g'(x)\cdot f(x)}{g(x)^2}$$

Example

s^{-1}

$$q(s) = \frac{s^2\cdot\left(1+1/s\right)}{1+s^2}$$

$$q'(s) = \frac{\left[\overbrace{2s\cdot\left(1+\frac{1}{s}\right) + s^2\left(0 + -1\cdot s^{-2}\right)}^{\text{derivative of } s^2\left(1+\frac{1}{s}\right)}\right]\cdot\left(1+s^2\right) - (0+2s)\cdot s^2\cdot\left(1+1/s\right)}{\left(1+s^2\right)^2}$$

or

$$q(s) = \frac{s^2 + s}{1 + s^2}$$

$$q'(s) = \frac{(2s+1)(1+s^2) - (0+2s)(s^2+s)}{(1+s^2)^2}$$

2. dy/dx Derivative Notation

If $y = f(x)$ is a function, the derivative of the function at a general point x can be written as either:

$f'(x)$ or $\frac{dy}{dx}$ $\frac{\text{"change in y"}}{\text{"change in x"}}$

$\boxed{\text{ALL THREE MEAN THE DERIVATIVE}}$ or $\frac{df}{dx}$

The value of the derivative at a specific value of x (e.g. $x=2$) would be written:

$$f'(2) \quad \underline{\text{or}} \quad \left.\frac{dy}{dx}\right|_{x=2} \quad \underline{\text{or}} \quad \left.\frac{df}{dx}\right|_{x=2}$$

3. <u>The Chain Rule for Derivatives</u>

- This is for taking derivatives of composite functions (i.e. where we have one function inside another function).

e.g. $f(x) = (2x+9)^{17}$

Built of: $h(x) = x^{17}$

$g(x) = 2x+9$

so: $f(x) = h(g(x))$.

$$f(x) = (\underbrace{2x+9}_{\text{inside function}})^{17}$$ ← outside function.

Statement of the Chain Rule

Suppose $f(x) = h(g(x))$.

Then:

$$f'(x) = h'(g(x)) \cdot g'(x)$$

derivative of outside

pure & unadulterated inside function

Suppose $z = h(y)$ and $y = g(x)$. Then:

$$\frac{dz}{dx} = \frac{dz}{dy} \cdot \frac{dy}{dx}.$$ Also the Chain Rule — good for applications.

Examples of the Chain Rule

① $f(x) = (2x+9)^{17}$

$$f'(x) = 17 \cdot (2x+9)^{16} \cdot 2$$

② $h(z) = \sqrt{1 + \sin^2(z)}$

$$h(z) = \left(1 + \sin^2(z) \right)^{1/2}$$

$$h'(z) = \frac{1}{2}\left(1 + \underbrace{\sin^2(z)}_{\sin(z)\cdot\sin(z)} \right)^{-1/2} \cdot \left(0 + 2\sin(z)\cos(z)\right)$$

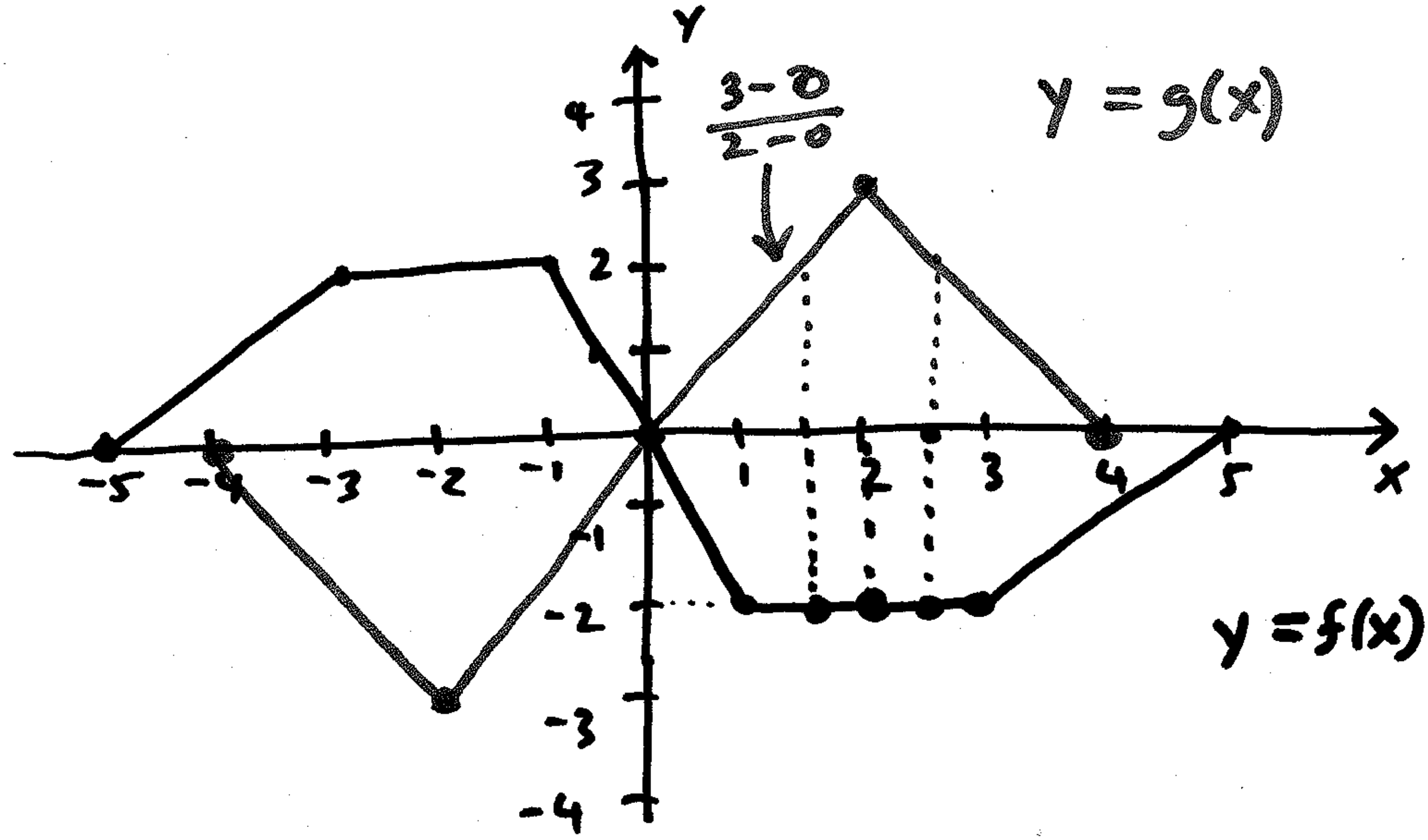
Deriv $= \cos(z)\cdot\sin(z) + \sin(z)\cdot\cos(z)$

③ $k(z) = \sin^2(z) = \left(\sin(z) \right)^2$

$$k'(z) = 2\left(\sin(z)\right)^1 \cdot \cos(z)$$

Example

Consider $f(x)$ and $g(x)$ defined by the graphs:



Let: $h(x) = f(x) \cdot g(x)$

$k(x) = f(x) / g(x)$

$p(x) = f(g(x))$.

Want: (a) $h'(1.5)$

(b) $k'(2.5)$

(c) $p'(1.5)$.

(a) $h(x) = f(x) \cdot g(x)$

want: $h'(1.5)$.

$$h'(1.5) = f'(1.5) \cdot g(1.5) + f(1.5) \cdot g'(1.5)$$

$$(0) \qquad (2) \qquad (-2) \qquad (3/2)$$

$$= 0 - 3$$

$$= -3$$

(b) $k(x) = \dfrac{f(x)}{g(x)}$ $\cancel{k'(1.5) = f'(1.5)g(1.5) - g'(1.5)f(1.5)}$

$$k'(2.5) = \frac{f'(2.5)\, g(2.5) - g'(2.5)\, f(2.5)}{g(2.5)^2}$$

$$= \frac{(0)(2) - (-3/2)(-2)}{(2)^2}$$

$$= -3/4$$

(c) $p(x) = f(g(x))$ $\quad p'(1.5) = f'(g(1.5))\, g'(1.5)$

$$p'(1.5) = f'(\,2\,)\ (3/2)$$

$$= (0)(3/2)$$

$$= 0.$$