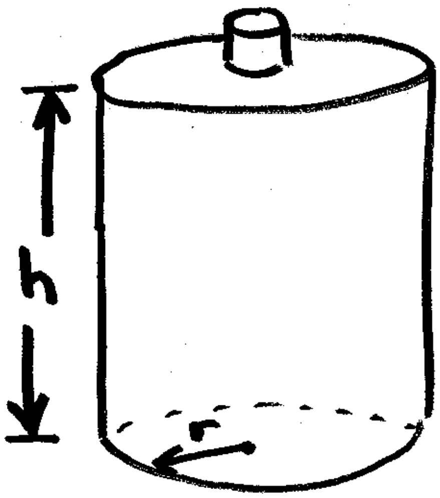
Outline

1. Finish bottle problem.

2. Short cut rules for derivatives.

3. Product and quotient rules.

1. Why Derivatives?

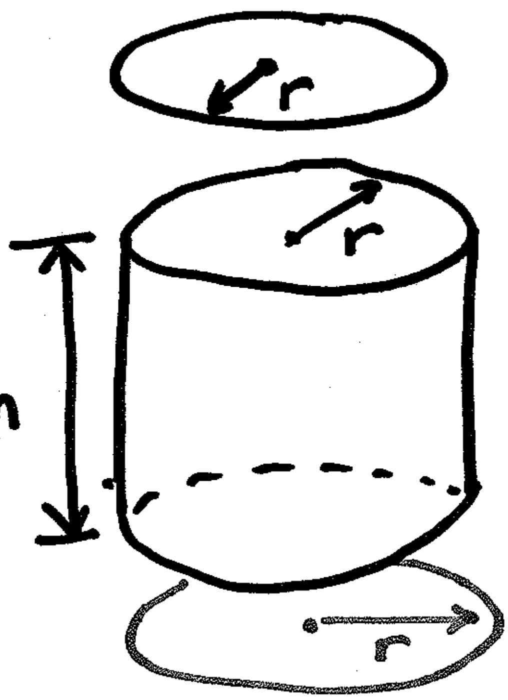


Volume has to be 500 ml (500 cubic centimeters).

Want surface area to be as low as possible.

Solution

Formula for surface area, S.



$$S = \pi r^2 + \pi r^2 + 2\pi r \cdot h$$

$$S = 2\pi r^2 + 2\pi r h$$

two variables

- Want to turn the formula for S into a formula with only one input variable (r).

- Write down a formula for volume and use the fact that the volume has to be 500.

$$\pi r^2 \cdot h = 500$$

$$h = \frac{500}{\pi r^2}$$

- Use this to replace "h" in formula for S:

$$\begin{aligned} S &= 2\pi r^2 + 2\pi r \cdot h \\ &= 2\pi r^2 + 2\pi r \cdot \frac{500}{\pi r^2} \\ &= 2\pi r^2 + \frac{1000}{r}, \quad r \neq 0. \end{aligned}$$

Plan: ① Calculate $S'(r)$.

② Set $S'(r) = 0$.

③ Solve for r.

Step ①: Calculate $S'(r)$

$$S(r) = 2\pi r^2 + \frac{1000}{r}$$

$$= 2\pi r^2 + 1000 \cdot r^{-1}$$

$$S'(r) = (2\pi) \cdot 2 r^1 + (1000) \cdot (-1) \cdot r^{-2}$$

$$= 4\pi r - \frac{1000}{r^2}$$

Step ②: Set $S'(r) = 0$

$$4\pi r - \frac{1000}{r^2} = 0$$

- Because max & min values of a function usually occur when derivative = 0.

<u>Step ③: Solve for r</u>

$$4\pi r - \frac{1000}{r^2} = 0$$

add $\frac{1000}{r^2}$ to both sides.

$$4\pi r = \frac{1000}{r^2}$$

multiply by r^2

$$4\pi r^3 = 1000$$

divide by 4π

$$r^3 = \frac{1000}{4\pi}$$

4.3

$r = \left(\frac{1000}{4\pi}\right)^{1/3} \approx$ [redacted] cm

$h = \frac{500}{\pi(\text{[redacted]})^2} \approx 8.61$ cm

4.3

2. Short Cut Rules for Derivatives

$f(x)$	$f'(x)$
$f(x) = K$ constant number.	$f'(x) = 0$
$f(x) = x^n$ e.g. $f(x) = x^3$	$f'(x) = n \cdot x^{n-1}$ e.g. $f'(x) = 3x^2$
$f(x) = K \cdot g(x)$ constant	$f'(x) = K \cdot g'(x)$
$f(x) = g(x) + h(x)$	$f'(x) = g'(x) + h'(x)$
$f(x) = g(x) - h(x)$	$f'(x) = g'(x) - h'(x)$
$f(x) = \sin(x)$	$f'(x) = \cos(x)$
$f(x) = \cos(x)$	$f'(x) = -\sin(x)$

3. Product and Quotient Rules

(a) Product Rule

$$f(x) = \underbrace{g(x) \bullet h(x)}_{\text{two functions multiplied.}}$$

$$\boxed{f'(x) = g(x) \cdot h'(x) + g'(x) \cdot h(x)}$$

(b) Quotient Rule

$$f(x) = g(x) / h(x) \left.\vphantom{\frac{g}{h}}\right\} \text{quotient of two functions}$$

$$\boxed{f'(x) = \frac{h(x) \cdot g'(x) - g(x) \cdot h'(x)}{h(x)^2} \qquad \frac{\text{low} \cdot d\,\text{high} - \text{high} \cdot d\,\text{low}}{\text{low} \cdot \text{low}}}$$

Example

$$m(z) = z^7 \cdot \left(3z^{1/4} + z^2\right)^{-1}$$

$$= z^7 \cdot \frac{1}{3z^{1/4} + z^2}$$

$$= \frac{z^7}{3z^{1/4} + z^2}$$

$$m'(z) = \frac{\left(3z^{1/4} + z^2\right) \cdot 7z^6 - z^7 \cdot \left(3\frac{1}{4}z^{-3/4} + 2z^1\right)}{\left(3z^{1/4} + z^2\right)^2}$$