Outline

1. Graph of derivative.
2. $f'(x)$ and the graph of $f(x)$.
3. Why derivatives?
4. Short cut rules for finding $f'(x)$.

1. Graph of Derivative $f'(x)$

- Slope of the tangent line to $y = f(x)$ gives the height of the derivative graph $y = f'(x)$.

- when we reach
  - top of a hill
  - bottom of a valley

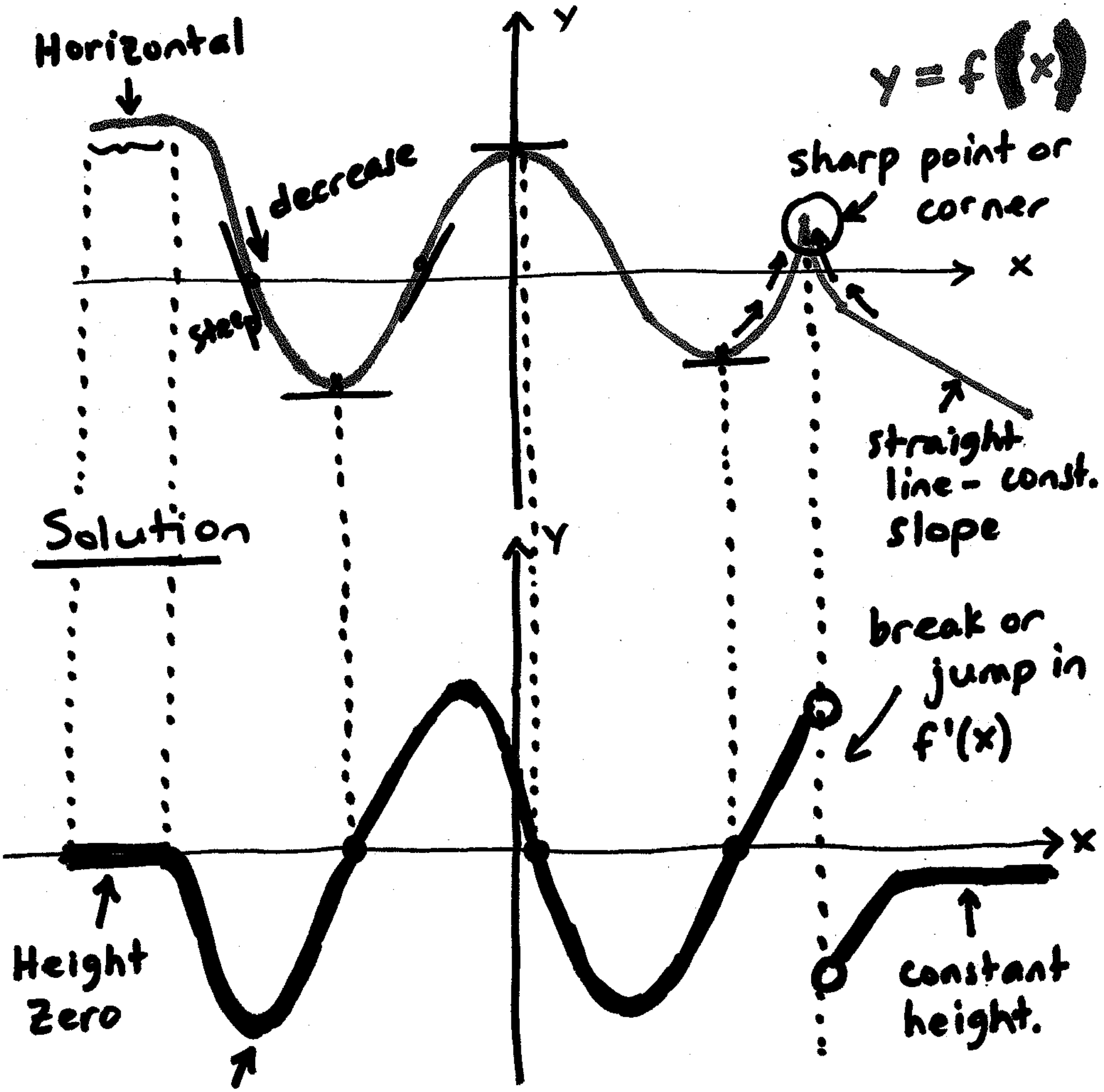
  $f'(x) = 0$, so the derivative graph will touch or cross the x-axis.

- when $f(x)$ is increasing, then $f'(x)$ is positive, $y = f'(x)$ has positive height.

- when $f'(x)$ is decreasing, then $f'(x)$ is negative, $y = f'(x)$ has negative height.

Example

Sketch a graph showing $y = f'(x)$.



steep negative slope of $f(x)$ means low negative height for $f'(x)$

2. $f'(x)$ and the Graph of $y = f(x)$

$f'(x)$	Behavior of $y = f(x)$
0	• Horizontal • Top of hill (maximum) • Bottom of valley (minimum)
+	Increasing (left to right)
−	Decreasing (left to right)
Break or jump	• Missing point on $f(x)$ • Sharp corner • Vertical asymptote

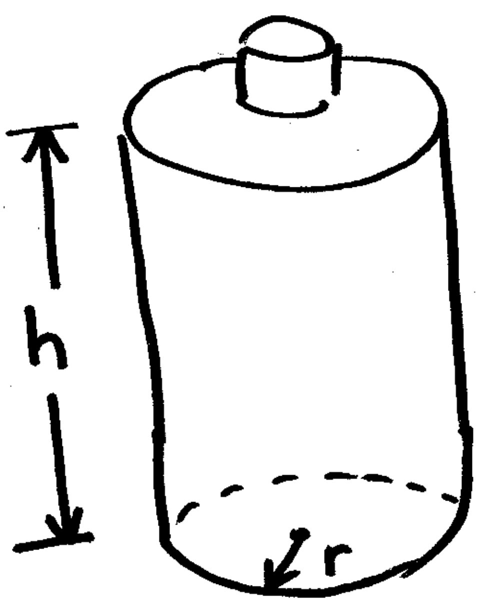
3. Why Derivatives

Example

How do you make a bottle to hold 500 ml (cubic centimeters) of water that uses as little plastic as possible?

Solution

h = height
r = radius.



Amount of plastic is the surface area of the bottle.

Goal: Find h, r so that volume = 500 ml, but surface area is as small as possible.

$S = $ surface area $= 2\pi r h + 2\pi r^2$