Solutions to Homework #4

Problems from Pages 80-83 (Section 2.1)

4. The slope of the tangent line is given by the derivative of y evaluated at the point $x = -1$. The calculation of this derivative is shown below.

$$\text{Slope} = \lim_{h \to 0} \frac{2(-1+h)^3 - 5(-1+h) - \left(2\cdot(-1)^3 - 5(-1)\right)}{h}$$

$$\text{Slope} = \lim_{h \to 0} \frac{6h - 3h^2 + h^3 - 5h}{h} = 6 - 5 = 1.$$

The x and y coordinates of the point of tangency are $(-1, 3)$ so the equation of the tangent line is:

$$y - 3 = x + 1.$$

14. The displacement of the particle is given by the equation: $s = t^2 - 8t + 18$.

(a) The average velocity over each time interval is given in the table below.

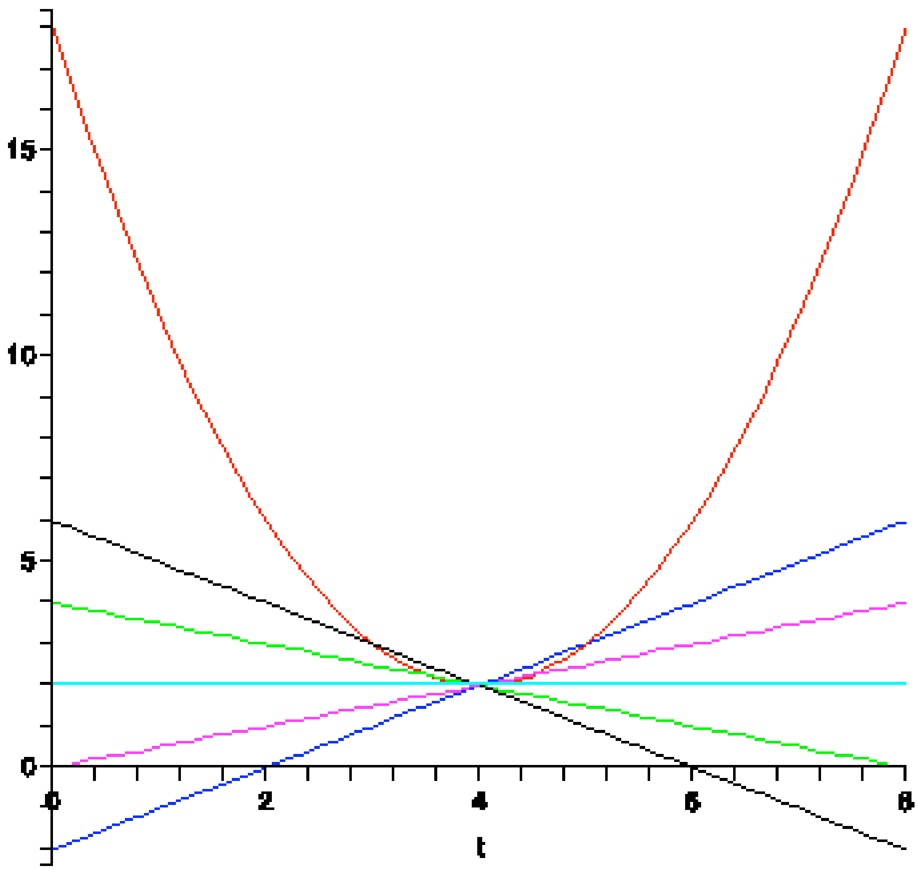
Part	Interval	Average velocity
(i)	[3, 4]	–1 m/s
(ii)	[3.5, 4]	–0.5 m/s
(iii)	[4, 5]	1 m/s
(iv)	[4, 4.5]	0.5 m/s

(b) The instantaneous speed when $t = 4$ is given by the derivative at $t = 4$. The calculation for the derivative is shown below.

$$\text{Speed} = \lim_{h \to 0} \frac{(4+h)^2 - 8(4+h) + 18 - \left(4^2 - 32 + 18\right)}{h} = \lim_{h \to 0} \frac{h^2}{h} = 0.$$

The instantaneous speed at $t = 4$ is 0 m/s.

(c) The graph with the secant lines is shown below.



16. The derivative is the slope of the tangent line. The x and y coordinates of the point of tangency are given by the x-coordinate of the point where the function is evaluated and the y-coordinate is the output value of the function.

(a) $y + 3 = 4\cdot(x - 5)$.

(b) The value of $f(4)$ is the y-coordinate of the point of tangency. This is $f(4) = 3$. The derivative is equal to the slope of the line joining the points (0, 2) and (4, 3). This gives:

$$f'(4) = \frac{3-2}{4-0} = \frac{1}{4}.$$

26. To calculate the derivative of $f(x) = \dfrac{x^2+1}{x-2}$ we will use the definition of the derivative and limits.

$$f'(a) = \lim_{h \to 0} \frac{\frac{(a+h)^2+1}{a+h-2} - \frac{a^2+1}{a-2}}{h} = \lim_{h \to 0} \frac{\frac{\left[(a+h)^2+1\right]\cdot(a-2) - (a^2+1)\cdot(a+h-2)}{(a+h-2)\cdot(a-2)}}{h}.$$

Simplifying this expression by FOILing and canceling terms gives:

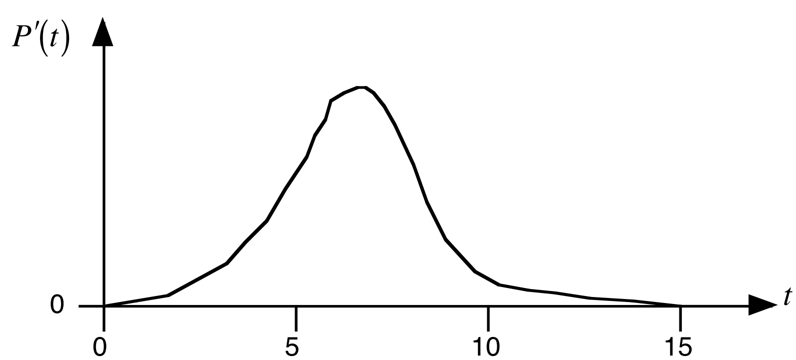
$$f'(a) = \lim_{h \to 0} \frac{a^2 - 4a + ah - 2h - 1}{(a+h-2)\cdot(a-2)} = \frac{a^2-4a-1}{(a-2)^2}.$$

Problems from Pages 91-94 (Section 2.2)

2. To get the values of the derivative at the various points, you can draw in a tangent line at each given *x*-value, find the *x* and *y* coordinates of points on each tangent line, and then calculate the slope of the line using these points. The results of doing this are summarized in the table given below.

Part	Derivative	Approximate value
(a)	$f'(0)$	−3
(b)	$f'(1)$	0
(c)	$f'(2)$	1.5
(d)	$f'(3)$	2
(e)	$f'(4)$	0
(f)	$f'(5)$	−1.2

12. A sketch of the derivative of the function *P*(*t*) is shown below.



The graph of the derivative has a peak when the population reaches half its eventual maximum value. This suggests that the population is growing fastest when it is at half its maximum level.

34. The graph of the function $f(x)$ is (d), the first derivative is (c), the second derivative is (b) and the third derivative is (a).

Problems from Pages 104-106 (Section 2.3)

36. The graph of the function $f(x)$ will have a horizontal tangent line when the derivative is equal to zero. The derivative of the function $f(x)$ is:

$$f'(x) = 3x^2 + 6x + 1.$$

Setting the derivative equal to zero and solving for x using the quadratic formula gives:

$$x = \frac{-6 \pm \sqrt{36 - 12}}{6} = -1 \pm \frac{\sqrt{6}}{3}.$$

52. The rate at which the water drains from the tank is given by the derivative of the function $V(t) = 5000(1 - t/40)^2$. This derivative is given by:

$$V'(t) = -250\left(1 - \tfrac{1}{40}t\right).$$

The values of the derivative at each of the given times are summarized in the table given below.

Part	t (minutes)	$V'(t)$ gallons per minute
(a)	5	–218.75
(b)	10	–187.5
(c)	20	–125
(d)	40	0

The water is flowing out the fastest when $t = 0$ and slowest when $t = 40$ minutes.

Problems from Pages 111-113 (Section 2.4)

44. **(a)** The derivative of $P(x) = F(x)\cdot G(x)$ is given by the product rule:

$$P'(x) = F'(x)\cdot G(x) + F(x)\cdot G'(x).$$

Estimating from the graphs, $F(2) = 3$, $G(2) = 2$, $F'(2) = 0$ and $G'(2) = 0.5$ so that $P'(2) = 1.5$.

(b) The derivative of $Q(x) = F(x)/G(x)$ is given by the quotient rule:

$$Q'(x) = \frac{F'(x) \cdot G(x) - G'(x) \cdot F(x)}{G(x)^2}.$$

Estimating from the graphs, $F(7) = 5$, $G(7) = 1$, $F'(7) = 0.25$ and $G'(7) = -2/3$ so that $Q'(7) = 43/12$.