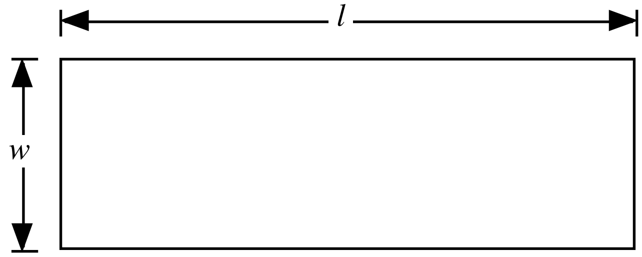
Solutions to Homework #2

Problems from Pages 8-10 (Section 1.1)

46. The rectangle in question is shown below with length l and width w.



The perimeter of the rectangle is: $P = 2l + 2w$.

On one hand, the area of the rectangle is given by $l \cdot w$ and on the other hand the area is 16. We can equate these and rearrange to make l the subject of the equation.

$$l = \frac{16}{w}.$$

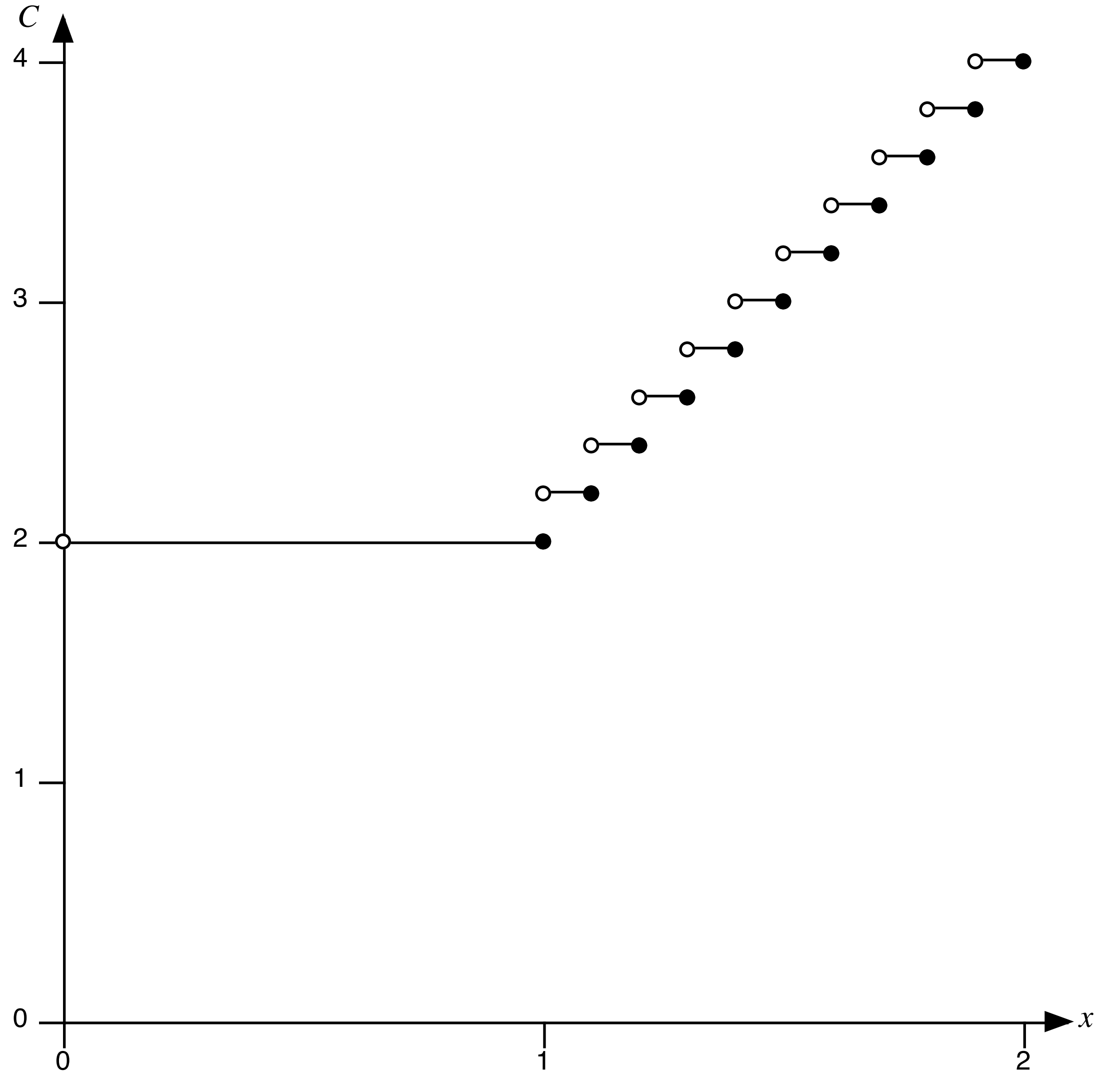
Next, we can use this to substitute for l in the equation for perimeter, giving:

$$P = 2l + 2w = 2\left(\frac{16}{w}\right) + 2w = \frac{32}{w} + 2w.$$

50. There are many ways to express the cost, C, as a function of x. One way to do this is as a function defined in pieces. (You can also write a more compact formula using either the floor or ceiling function.)

$$C(x) = \begin{cases} 2 & ,0 < x \le 1 \\ 2.2 & ,1.1 < x \le 1.2 \\ \vdots & \vdots \\ 4 & ,1.9 < x \le 2 \end{cases}.$$

The graph of this function looks like a series of steps, as shown below.



Problems from Pages 21-24 (Section 1.2)

6. **(a)** The slope of the function is 0.02. This means that each year the average surface temperature of the world goes up by 0.02°C. The intercept of the function is 8.50°C. This means that when $t = 0$ (i.e. the year 1900), the average surface temperature of the world was 8.50°C.

(b) In the year 2100, the value of t is $t = 200$. Plugging this into the equation gives:

$$T = 0.02(200) + 8.50 = 12.5°\text{C}.$$

12. **(a)** We can summarize the information given in the problem with a table like the one shown below. We will use x to represent the number of chairs made in a day and C to represent the cost in dollars.

x	100	300
C	2200	4800

To find the slope of the linear function calculate "change in *C* over change in *x*:"

$$\text{Slope} = \frac{\Delta C}{\Delta x} = \frac{4800 - 2200}{300 - 100} = 13.$$

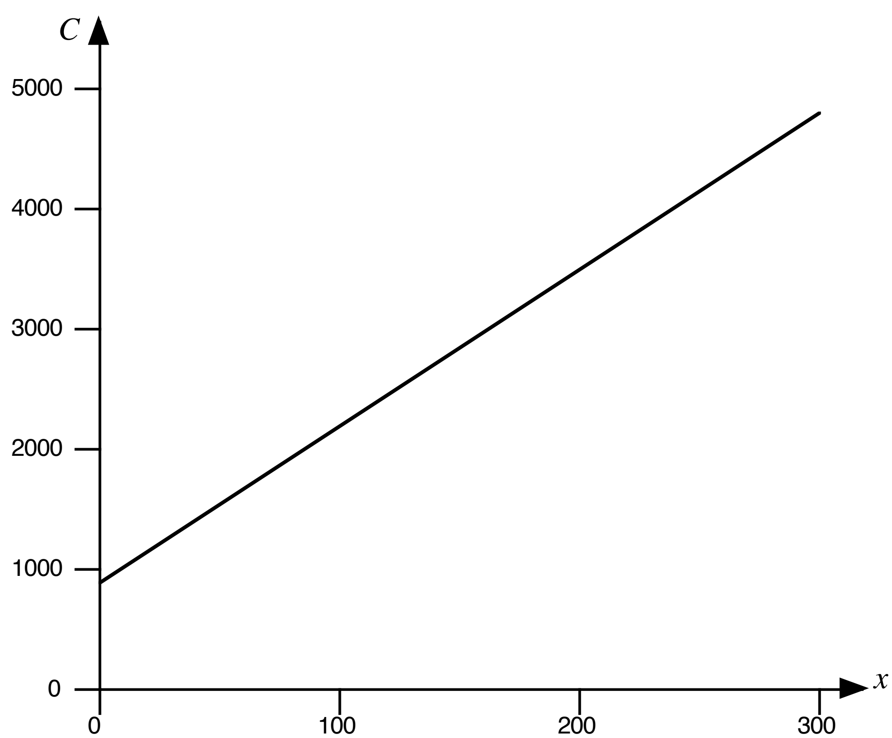
To find the intercept, *b*, we will plug the point (x, C) = (100, 2200) into the formula:

$$C = 13 \cdot x + b,$$

to calculate *b*. Doing this gives $b = 900$, so that the linear function expressing *C* as a function of *x* is:

$$C = 13 \cdot x + 900.$$

The graph of this function is shown below.



(b) The slope of the graph is 13. This means that the manufacture of each chair adds \$13 to the daily cost of operating the factory. (In economics, the figure of \$13 is called the *marginal cost*.)

(c) The intercept of the graph is 900. This means that even when no chairs are manufactured, it costs \$900 per day to operate the factory. (In economics, the figure of \$900 is called the *fixed cost*.)

38. **(a)** $(f \circ g)(x) = 1 - \left(\frac{1}{x}\right)^3$. The domain consists of all real numbers except $x = 0$.

(b) $(g \circ f)(x) = \frac{1}{1 - x^3}$. The domain consists of all real numbers except $x = 1$.

(c) $(f \circ f)(x) = 1 - \left(1 - x^3\right)^3$. The domain consists of all real numbers.

(d) $(g \circ g)(x) = x$. The domain consists of all real numbers except $x = 0$.

42. **(a)** $(f \circ g)(x) = \sqrt{2\left(x^2 + 1\right) + 3}$. The domain consists of all real numbers.

(b) $(g \circ f)(x) = \left(\sqrt{2x + 3}\right)^2 + 1$. The domain consists of all real numbers $x \geq -1.5$.

(c) $(f \circ f)(x) = \sqrt{2\sqrt{2x + 3} + 3}$. The domain consists of all real numbers $x \geq -1.5$.

(d) $(g \circ g)(x) = \left(x^2 + 1\right)^2 + 1$. The domain consists of all real numbers.

52. **(a)** $f(g(1)) = f(6) = 5$.

(b) $g(f(1)) = g(3) = 2$.

(c) $f(f(1)) = f(3) = 4$.

(d) $g(g(1)) = g(6) = 3$.

(e) $(g \circ f)(3) = g(f(3)) = g(4) = 1$.

(f) $(f \circ g)(6) = f(g(6)) = g(3) = 4$.

54. **(a)** We will assume at the time $t = 0$ that the radius, r, is also zero. The function is then: $r(t) = 2t$.

(b) Since the volume of a sphere is given by the formula $V = \frac{4\pi}{3} r^3$, finding the composition of this function with the function $r(t)$ from Part (a) gives:

$$V(t) = V(r(t)) = \frac{4\pi}{3}(r(t))^3 = \frac{4\pi}{3}(2t)^3 = \frac{32\pi}{3}t^3.$$

Problems from Pages 33-35 (Section 1.3)

2. **(a)** The average velocity of the arrow over each of the intervals is given in the table below.

Problem	Interval	Average velocity (m/s)
(i)	[1, 2]	55.51
(ii)	[1, 1.5]	55.925
(iii)	[1, 1.1]	56.257
(iv)	[1, 1.01]	56.3317
(v)	[1, 1.001]	56.33917

(b) The values of the average velocity in the table from Part (a) seem to be getting closer and closer to some number that is a little more than 56.33, so I would guess that the instantaneous velocity is approximately 56.34 m/s when t = 1.

4. **(a)** $\lim_{x \to 0} f(x) = 3$.

(b) $\lim_{x \to 3^-} f(x) = 4$

(c) $\lim_{x \to 3^+} f(x) = 2$

(d) $\lim_{x \to 3} f(x)$ does not exist because the left hand and right hand limits are not equal.

(e) $f(3) = 3$.