Solutions to Homework #1

Problems from Pages 8-10 (Section 1.1)

2. **(a)** Reading the y-values from the graph we can say that $f(-4) = -2$ and $g(3) = 4$.

(b) The values of x at which $f(x) = g(x)$ are the x-coordinates where the two graphs cross. These are $x = -2$ and $x = 2$.

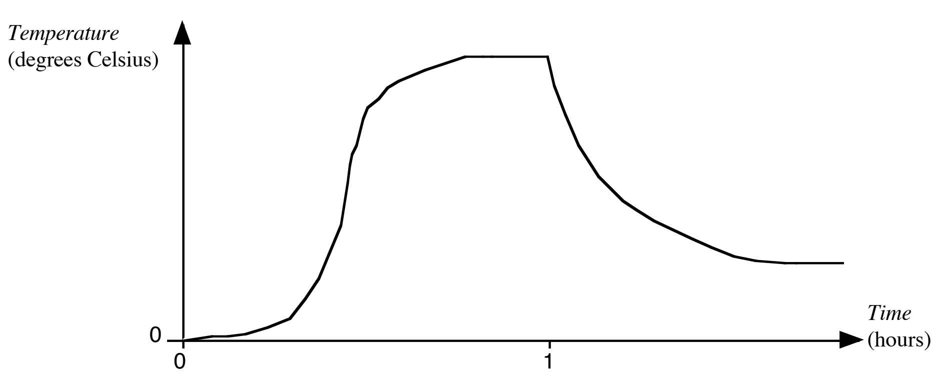
(c) The solutions of the equation $f(x) = -1$ are the x-coordinates of the points on the blue graph where the y-coordinate is equal to -1. These are $x = -3$ and $x = 4$.

(d) The graph of $y = f(x)$ is increasing (rising when read from left to right) on the interval $(-4, 0)$. Note that the end-points of the interval are not included.

(e) The domain of f certainly includes the interval $(-4, 4)$. The graph on page 8 does not indicate whether f is defined at the end-points or not. If it is then $[-4, 4]$ would also be an acceptable answer. The range of f certainly includes the interval $(-2, 3]$, and if f is defined at the endpoints of the graph then $[-2, 3]$ would also be an acceptable answer.

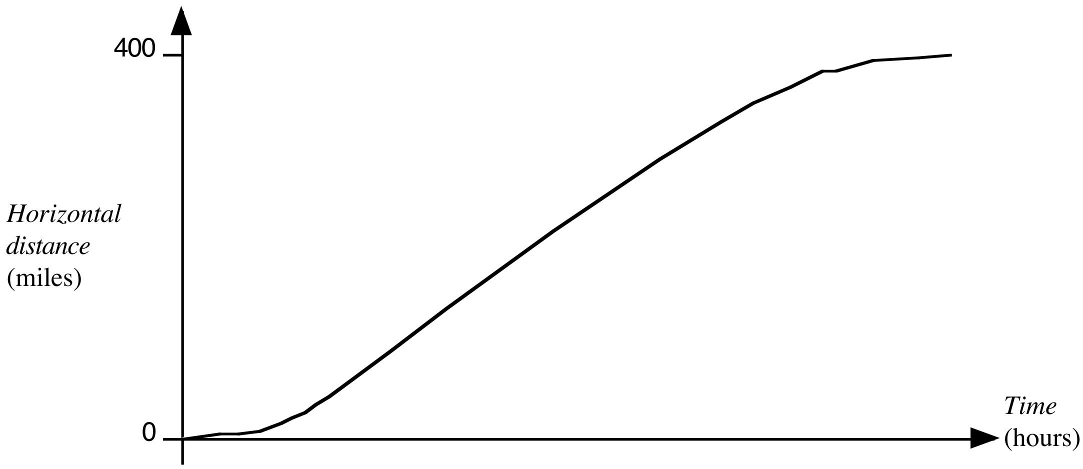
(f) The domain of g certainly includes the interval $(-4, 3)$. The graph on page 8 does not indicate whether g is defined at the end-points or not. If it is then $[-4, 3]$ would also be an acceptable answer. The range of g certainly includes the interval $[0.5, 4)$, and if f is defined at the endpoints of the graph then $[0.5, 4]$ would also be an acceptable answer.

14. There are many possible answers to this problem. One potential solution is shown in the graph given below. Note where this graph is increasing, decreasing and where it flattens out (horizontal asymptote). Any correct solution will share these features.

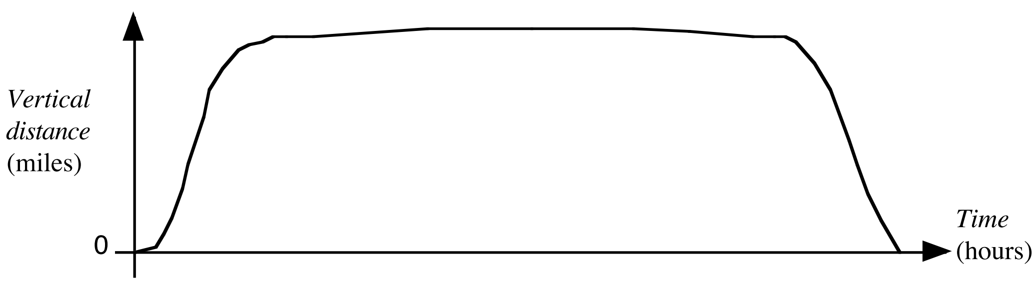


16. As with the previous problem, many graphs will represent potentially correct answers to this problem. If you know a lot about how jets normally fly from one airport to another, you can probably make your answer much more elaborate (and accurate) than the graphs given below.

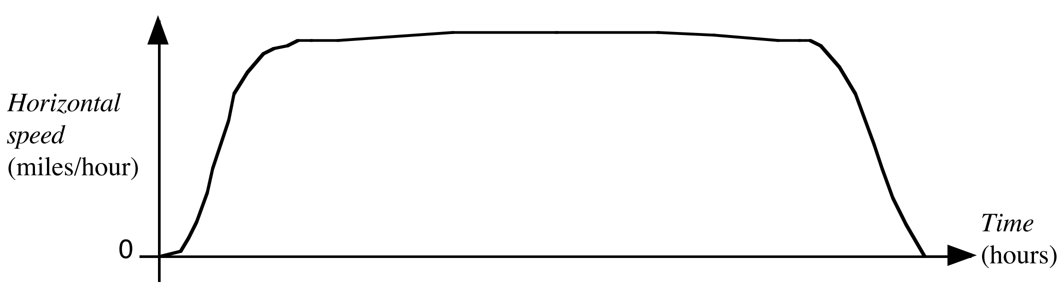
(a) I assumed that the jet accelerated at the start of the flight, maintained a steady speed, and then decelerated at the end of the flight.



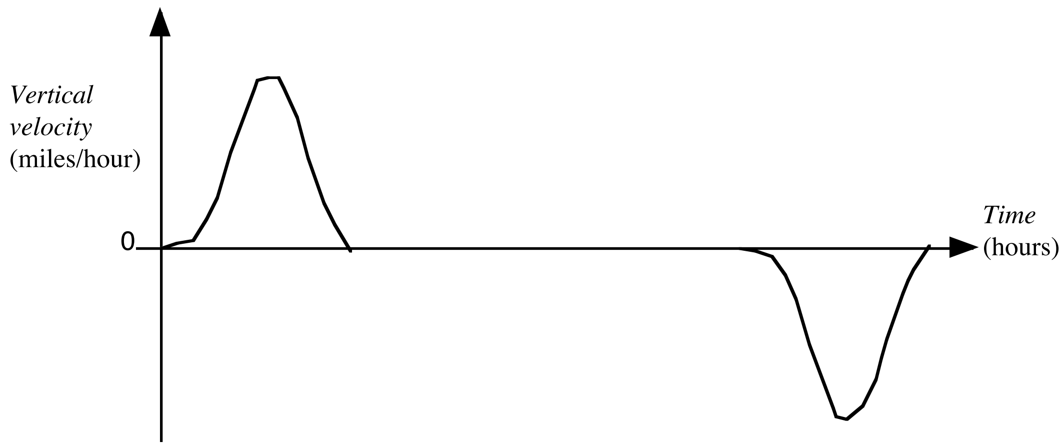
(b) I assumed that the jet cruised at a constant altitude for most of the trip with a rapid climb at the beginning and a rapid dive at the end.



(c) I assumed that the jet accelerated at the start of the flight, maintained a steady speed, and then decelerated at the end of the flight.



(d) I assumed that the jet cruised at a constant altitude for most of the trip with a rapid climb at the beginning and a rapid dive at the end. Positive velocity represents the jet gaining altitude and negative velocity represents the jet losing altitude.



20. The difference quotient of $f(x) = x^3$ is as follows.

$$\frac{f(a+h)-f(a)}{h} = \frac{(a+h)^3 - a^3}{h} = \frac{a^3 + 3a^2h + 3ah^2 + h^3 - a^3}{h} = 3a^2 + 3ah + h^2.$$

22. The difference quotient of $f(x) = \dfrac{x+3}{x+1}$ is as follows.

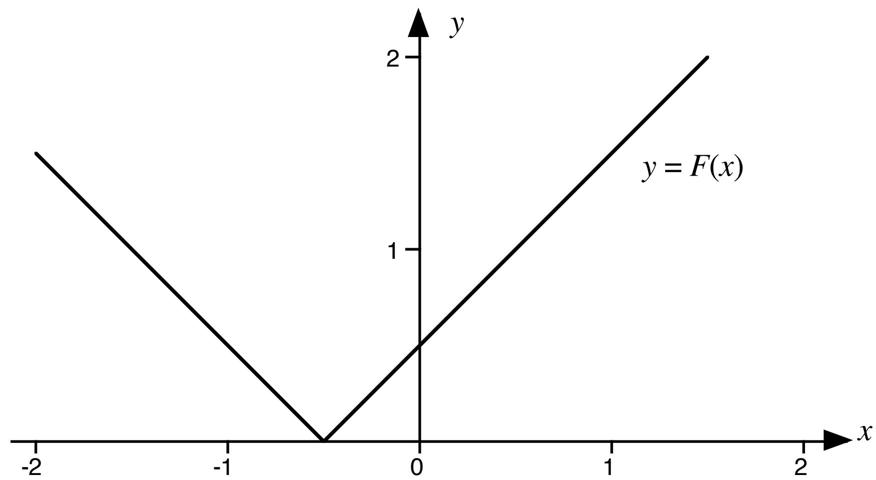
$$\frac{f(x)-f(1)}{x-1} = \frac{\frac{x+3}{x+1} - \frac{4}{2}}{x-1} = \frac{\frac{x+3-2(x+1)}{x+1}}{x-1} = \frac{-x+1}{(x+1)(x-1)} = \frac{-1}{x+1}.$$

24. The domain of $f(x)$ will include all real numbers x with the exception of those that make the denominator of $f(x)$ equal to zero. We can locate those x-values by factoring the denominator of $f(x)$.

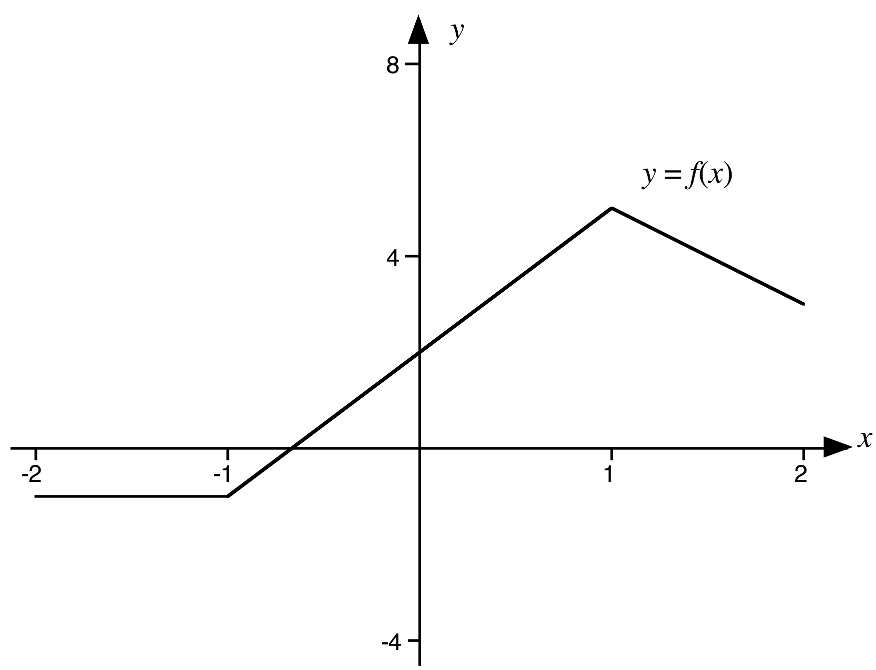
$$\text{Denominator} = x^2 + 3x + 2 = (x+1)(x+2).$$

So the x-values that will make the denominator of $f(x)$ equal zero are $x = -1$ and $x = -2$. The domain of $f(x)$ consists of all real numbers except $x = -1$ and $x = -2$.

34. There are no fractions or square roots in the formula for $F(x)$ so there is nothing to restrict the x-values that can be substituted into this function. As a result, the domain of $F(x)$ consists of all real numbers. The graph of $y = F(x)$ is shown on the next page.

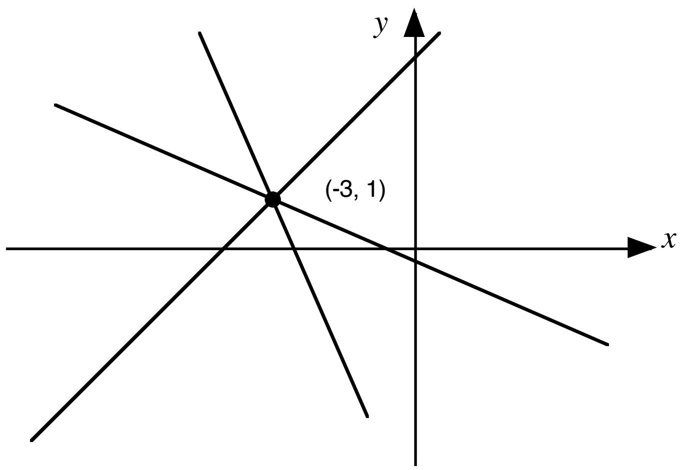


40. There are no x-values that are left out of the piecewise definition of the function $f(x)$ here so it is okay to plug any real number into the function as x. This means that the domain of $f(x)$ consists of all real numbers. The graph of $y = f(x)$ is shown below.



Problems from Pages 21-24 (Section 1.2)

2. Each linear function of the form $f(x) = 1 + m(x+3)$ passes through the point (–3, 1). A sketch showing several functions (with different values of m) follows on the next page.



4. The formula for the first graph will have the form:

$$f(x) = k \cdot (x-3)^2,$$

as the graph shows only one root, and it has multiplicity 2. To determine the constant k we will use the point (4, 2) in this formula:

$$2 = k \cdot (4-3)^2,$$

to obtain $k = 2$. This gives the formula for $f(x)$ as:

$$f(x) = 2 \cdot (x-3)^2.$$

Unfortunately, the roots of the second graph are not labeled so we will have to use a different strategy to find a formula for it. The generic formula for a quadratic function is:

$$y = ax^2 + bx + c,$$

where a, b and c are all constants. To determine the value of c we can use the point (0, 1) in this equation. Plugging in $x = 0$ and $y = 1$ gives:

$$1 = a \cdot 0^2 + b \cdot 0 + c$$

so that $c = 1$. To find the values of a and b we can plug the two remaining points into the formula for y and then rearrange so that we get a system of linear equations for a and b. Doing this gives:

$$4a - 2b = 1$$

$$a + b = -3.5.$$

Solving this pair of linear equations gives $a = -1$ and $b = -2.5$ so that the final formula for the quadratic is:

$$y = -x^2 - 2.5x + 1.$$