Solutions to Homework #10

Problems from Pages 232-236 (Section 4.5)

40. In this problem, we will let x represent the number of \$10 price rises that the apartment manager makes.

If the manager makes x rises, then the monthly rent, $r(x)$, for each apartment will be:

$$r(x) = 800 + 10x.$$

On the other hand, then number of units, $n(x)$, occupied goes down by one for every \$10 price rise, so that:

$$n(x) = 100 - x.$$

The revenue, $R(x)$, that the manager collects will be the product of these two functions, i.e.:

$$R(x) = r(x)\cdot n(x) = (800 + 10x)(100 - x) = 80000 + 200x - 10x^2.$$

To find the value of x that will maximize the revenue we will calculate the derivative of $R(x)$, set this derivative equal to zero and then solve for x. Doing that gives:

$$R'(x) = 200 - 20x = 0,$$

so that $x = 10$. This means that the manager should raise rent by \$100 from \$800 to \$900.

To check that this is the global maximum we also need to consider the endpoints $x = 0$ and $x = 100$. To see which is the global maximum we will calculate $R(x)$ at each of the three points.

x	R(x)	Comment
0	80000	
10	81000	Global maximum
100	0	

So, the manager should set the rent at \$900 per month.

50. In this problem, we will make x the distance from the person to the wall in feet. Then:

$$\theta(x) = \tan^{-1}\left(\frac{h+d}{x}\right) - \tan^{-1}\left(\frac{d}{x}\right),$$

where h and d are constants (so x is the only variable in this function).

To find the value of x that maximizes θ, we will take the derivative of θ with respect to x, set the derivative equal to zero and solve for x. Doing this gives:

$$\frac{d\theta}{dx} = \frac{1}{1+\left(\frac{h+d}{x}\right)^2} \cdot \frac{-(h+d)}{x^2} - \frac{1}{1+\left(\frac{d}{x}\right)^2} \cdot \frac{-d}{x^2} = 0.$$

Simplifying this expression eventually gives:

$$\frac{h^2 d + hd^2 - hx^2}{\left[x^2 + (h+d)^2\right]\left(x^2 + d^2\right)} = 0,$$

so that by only considering the numerator, we get:

$$x^2 = hd + d^2.$$

In the context of this problem, we will assume that $x > 0$ so that the value of x that maximizes θ will be the positive root:

$$x = \sqrt{hd + d^2}.$$

Problems from Pages 260-261 (Section 5.1)

2. In this problem, the width of each rectangle is given by $\Delta x = (12 - 0)/6 = 2$.

(a) **(i)** The left-hand Riemann sum will be:

$$2\cdot(f(0) + f(2) + f(4) + f(6) + f(8) + f(10)) \approx 2\cdot(9 + 8.8 + 8.2 + 7.3 + 5.9 + 4.1) = 86.6$$

(ii) The right-hand Riemann sum will be:

$$2\cdot(f(2) + f(4) + f(6) + f(8) + f(10) + f(12)) \approx 2\cdot(8.8 + 8.2 + 7.3 + 5.9 + 4.1 + 1) = 70.6$$

(iii) The midpoint sum will be:

$$2\cdot(f(1) + f(3) + f(5) + f(7) + f(9) + f(11)) \approx 2\cdot(8.9 + 8.5 + 7.8 + 6.6 + 5.1 + 2.8) = 79.4$$

(b) Since the function $f(x)$ is decreasing, the left-hand Riemann sum will give an overestimate of the true value of the area under the curve.

(c) Since the function $f(x)$ is decreasing, the right-hand Riemann sum will give an underestimate of the true value of the area under the curve.

(d) Of the three sums calculated in Part (a), the midpoint sum is probably the best estimate of the area under the curve. This is because the areas that the midpoint sum leaves out are probably pretty close to the small extra areas that the midpoint sum includes.

4. In this problem, the width of each rectangle is given by $\Delta x = (5 - 0)/5 = 1$.

(a) The right-hand Riemann sum will be:

$$1\cdot(f(1) + f(2) + f(3) + f(4) + f(5)) \approx 1\cdot(24 + 21 + 16 + 9 + 0) = 70$$

Since the function $f(x)$ is decreasing, the right-hand Riemann sum will give an underestimate of the true value of the area under the curve.

(b) The left-hand Riemann sum will be:

$$1\cdot(f(0) + f(1) + f(2) + f(3) + f(4)) \approx 1\cdot(25 + 24 + 21 + 16 + 9) = 95$$

Since the function $f(x)$ is decreasing, the left-hand Riemann sum will give an overestimate of the true value of the area under the curve.

8. In this problem, the width of each rectangle beneath the velocity time graph will be given by the time interval between the entries in the table. In each case, this is 12 seconds.

(a) The left-hand Riemann sum will be:

$$L = 12\cdot(30 + 28 + 25 + 22 + 24) = 1548 \text{ feet.}$$

(b) The right-hand Riemann sum will be:

$$L = 12\cdot(28 + 25 + 22 + 24 + 27) = 1512 \text{ feet.}$$

(c) It is impossible to say with any confidence that either the left-hand or right-hand Riemann sum is an upper or a lower estimate for the distance. This is because velocity (as given in the table) is not a strictly increasing or strictly decreasing function of time, t. Only when the function is strictly increasing or strictly decreasing over an interval an we predict whether the left or right-hand Riemann sum will be an over or under estimate of the true area under the curve.

10. As the velocity of the space shuttle is always increasing, a left-hand sum will give an underestimate for the distance, whereas a right-hand sum will give an over estimate. The "best" estimate will be the average of these two.

LH sum = (0)(10) + (185)(5) + (319)(5) + (447)(12) + (742)(27) + (1325)(3)

= 31893 ft/s.

RH sum = (185)(10)+(319)(5)+(447)(5) + (742)(12) + (1325)(27) + (1445)(3)

= 54694 ft/s.

"Best" estimate = 43293.5 ft/s.

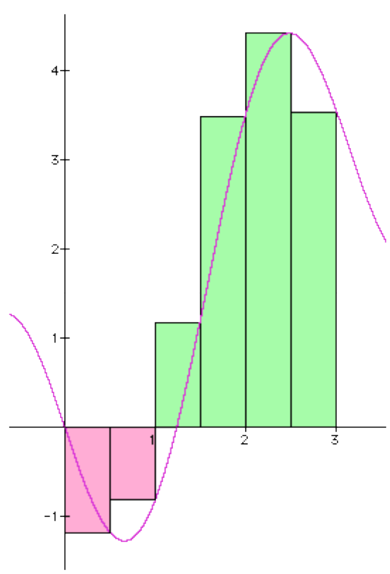
Problems from Pages 272-274 (Section 5.2)

4. In this problem $f(x) = x - 2\sin(2x)$ and $\Delta x = 0.5$.

(a) Right hand sum $= 0.5\cdot[f(0.5) + f(1) + f(1.5) + f(2) + f(2.5) + f(3)]$

≈ 5.353254

The right hand sum represents the areas of the green rectangles in the diagram shown below minus the areas of the pink rectangles.



(b) Right hand sum $= 0.5\cdot[f(0.25) + f(0.75) + f(1.25) + f(1.75) + f(2.25) + f(2.75)]$

≈ 4.458461

8. When we divide the interval [−3, 3] into six subintervals, the width of each will be $\Delta x = 1$.

(a) Using the right endpoints to approximate the integral we get:

$$\int_{-3}^{3} g(x)dx \approx g(-2) + g(-1) + g(0) + g(1) + g(2) + g(3)$$

$$\approx 1 - 0.5 - 1.5 - 1.5 - 0.5 + 2.5$$

$$= -0.5.$$

(b) Using the left endpoints to approximate the integral we get:

$$\int_{-3}^{3} g(x)dx \approx g(-3) + g(-2) + g(-1) + g(0) + g(1) + g(2)$$

$$\approx 2 + 1 - 0.5 - 1.5 - 1.5 - 0.5$$

$$= -1.$$

(c) Using the midpoints to approximate the integral we get:

$$\int_{-3}^{3} g(x)dx \approx g(-2.5) + g(-1.5) + g(-0.5) + g(0.5) + g(1.5)+g(2.5)$$

$$\approx 1.5 + 0 - 1 - 1.75 - 1 +0.5$$

$$= -1.75.$$

10. When we divide the interval [0, 6] into three subintervals, the width of each will be $\Delta x = 2$.

(a) Using the right endpoints to approximate the integral we get:

$$\int_{0}^{6} f(x)dx \approx 2\cdot[f(2) + f(4) + f(6)]$$

$$\approx 2\cdot[8.3 + 2.3 - 10.5]$$

$$= \quad 0.2.$$

(b) Using the left endpoints to approximate the integral we get:

$$\int_0^6 f(x)dx \approx 2\cdot[f(0) + f(2) + f(4)]$$

$$\approx 2\cdot[9.3 + 8.3 + 2.3]$$

$$= 39.8.$$

(c) Using the midpoints to approximate the integral we get:

$$\int_0^6 f(x)dx \approx 2\cdot[f(1) + f(3) + f(5)]$$

$$\approx 2\cdot[9.0 + 6.5 - 7.6]$$

$$= 15.8.$$

14. When we divide the interval [1, 5] into three subintervals, the width of each will be $\Delta x = 1$.

$$\int_1^5 x^2 \cdot e^{-x} \cdot dx \approx 1\cdot[(1.5)^2 e^{-1.5} + (2.5)^2 e^{-2.5} + (3.5)^2 e^{-3.5} + (4.5)^2 e^{-4.5}]$$

$$\approx 1.6099.$$