Recitation Handout 6: The Law of Forgetting

The specific learning goals of this activity are for you to:

- Convert a collection of observations into a set of numerical data.
- Use your data to calculate the rate of change of a function.
- Create a graph showing the relationship between the rate of forgetting and the amount of information remembered.
- Create a formula to express this relationship symbolically.
- Use your formula to create a differential equation.
- Gain insight into one process that can be used to create a differential equation to describe a psychological phenomenon.

In today's recitation you will be able to organize the data on remembering the list of words and synthesize it into a differential equation that defines a function by specifying its rate of change.

In lecture on Wednesday we will use your results to find a formula to describe the amount of information you retain as time passes.

Why have we been trying to remember this list of words?


Figure 1: Hermann Ebbinghaus (1850-1909).

The experiment that you have completed in this lab is based upon the work[1] of the German experimental psychologist, Hermann Ebbinghaus (1850-1909).

Ebbinghaus received his Ph.D. from the University of Bonn in 1873 and soon thereafter became an assistant professor at the Friedrich-Wilhelm University in Berlin.

At the time in which Ebbinghaus worked, European psychology was well underway, due to the pioneering efforts of individuals such as Hermann von Helmholtz, Gustav Fechner and Franz Brentano[2]. In the latter part of the nineteenth century, psychology grew from one department (at the University of Leipzig, founded in 1879) to an extensive field of study that included departments in many leading European universities.

The single figure who probably did more to establish psychology as a separate, experimental science with its own programs, methods and aims, was Wilhem Wundt.

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[1] Most notably: Ebbinghaus, H. 1885. *Memory: A Contribution to Experimental Psychology*. New York: Teachers College Press.

[2] For a discussion of the contributions of these individuals to the establishment of psychology, see Chapter 5 of: Gardner, H. 1985. *The Mind's New Science. A History of the Cognitive Revolution*. New York: Basic Books.

Wundt believed that psychology was best approached through introspection – by observers (or witnesses) who had been meticulously trained to observe the purely mental aspects of their own experiences. In Wundt's studies, trained observers carefully examined their own sensations and reported their observations as objectively as they possibly could. In such a context, "objectivity" usually meant that the observer did his or her best to report the thoughts and sensations that they experienced without trying to interpret (or ascribe) any meaning to them[3].

Although Wundt was a very productive and highly respected scholar, not everyone embraced his view of how psychology should be done. Instead of focusing on introspection, Ebbinghaus devised experiments that produced mathematical relationships between externally observable inputs (such as time) and externally observable outputs (number of words correctly recalled). In Ebbinghaus' work, it was the proficiency of a human at the experimental task that was the object of study, rather than the feelings, thoughts and sensations experienced by a human while completing the task.

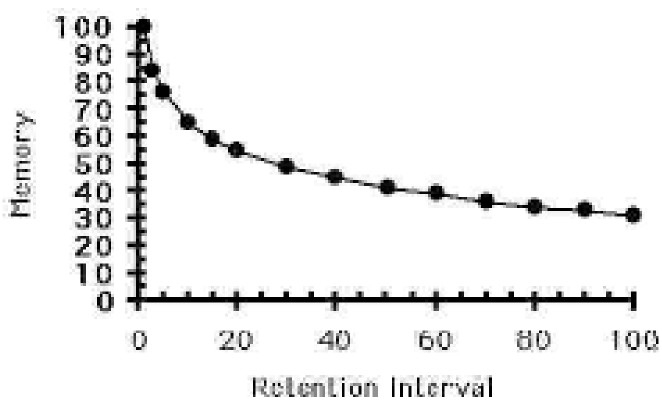

Figure 2: The Forgetting Curve.

Ebbinghaus' most famous work was on memory and tried to measure and describe the formation of the mental associations that underlie recollection of memories. (One part of this study was the experiment that you performed with the lists of "nonsense words" to generate the characteristic "forgetting curve.") Using himself as an experimental subject, Ebbinghaus devised about 2,300 three-letter "nonsense words" and systematically set about attempting to learn (and forget) them. He measured the rate at which his recollection of the words decayed, and from these measurements, created a mathematical formula that is sometimes called the "forgetting curve."

In today's recitation (and the lecture on Wednesday) you will recreate Ebbinghaus' forgetting curve using the ideas of differential equations.

Collating Your Results

In the first part of the recitation you will organize the information that you have generated over the course of the last few days to create a set of numerical data that we can analyze mathematically.

1. Get your white envelope and your yellow envelope from your recitation instructor. If you don't have a white and yellow envelope, you will need to find someone who does and ask them if it is okay for you to work with them today.

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[3] Johnson-Laird, P. N. and P. C. Wason. Eds. 1977. *Thinking: Readings in Cognitive Science*. Cambridge UK: Cambridge University Press.

2. Open the yellow envelope and get out the pieces of paper that you wrote your lists of words on. Sort them into order from the one that you completed first to the one you completed last.

3. Open the white envelope so that you have the list of words that you initially tried to memorize on Wednesday, February 25. Use this list of words to score each of your lists, noting the number of words that you got correct each time you wrote down the list.

4. Look at the date and time that you wrote on the list of words from the white envelope. Using this time and date as your reference. For each of the lists from the yellow envelope, figure out how much time had passed since you first learned the list of words. Write the elapsed time (expressed in units of days, with 1 hour being 1/24 of one day, etc.) on each of the lists of words that you made.

5. Each of the lists that came from the yellow envelope should now have a time (in days) and a number of words remembered completely correctly. Transfer these numbers (in order) to the table given below.

Elapsed time (in days)	Number of words remembered correctly

Table 1.

Creating Rates of Change

The entries in Table 1 form time intervals. In this part of the recitation, your objective will be to find the rate at which you forgot words during these time intervals and record these rates in a table.

6. Use the entries from the "Number of words remembered correctly" column of Table 1 to complete the first two columns of Table 2 (below). There is a short sample table immediately below to show you how to transfer the numbers from Table 1 to Table 2.

Sample Table 1:

Elapsed time (in days)	Number of words remembered correctly
0	10
1/24	7
1/3	5
1	4

Sample Table 2:

Number of words remembered correctly at start of interval	Number of words remembered correctly at end of interval (x)	Number of words forgotten during interval	Length of time interval (days)	Rate of change (y)
10	7	3	1/24 – 0 = 1/24	
7	5	2	1/3 – 1/24 = 7/24	
5	4	1	1 – 1/3 = 2/3	

Now use the numbers you have recorded in Table 1 to fill in the blank version of Table 2 on the next page.

Number of words remembered correctly at start of interval	Number of words remembered correctly at end of interval (x)	Number of words forgotten during interval	Length of time interval (days)	Rate of change (y)

Table 2.

7. Now complete the last column in Table 2 by multiplying the number of words forgotten by −1 and then dividing by the length of the time interval. (The reason for multiplying by −1 is because when words are forgotten, the number of words remembered decreases so the rate of change should be negative.) Sample Table 2 is shown below to illustrate the calculation.

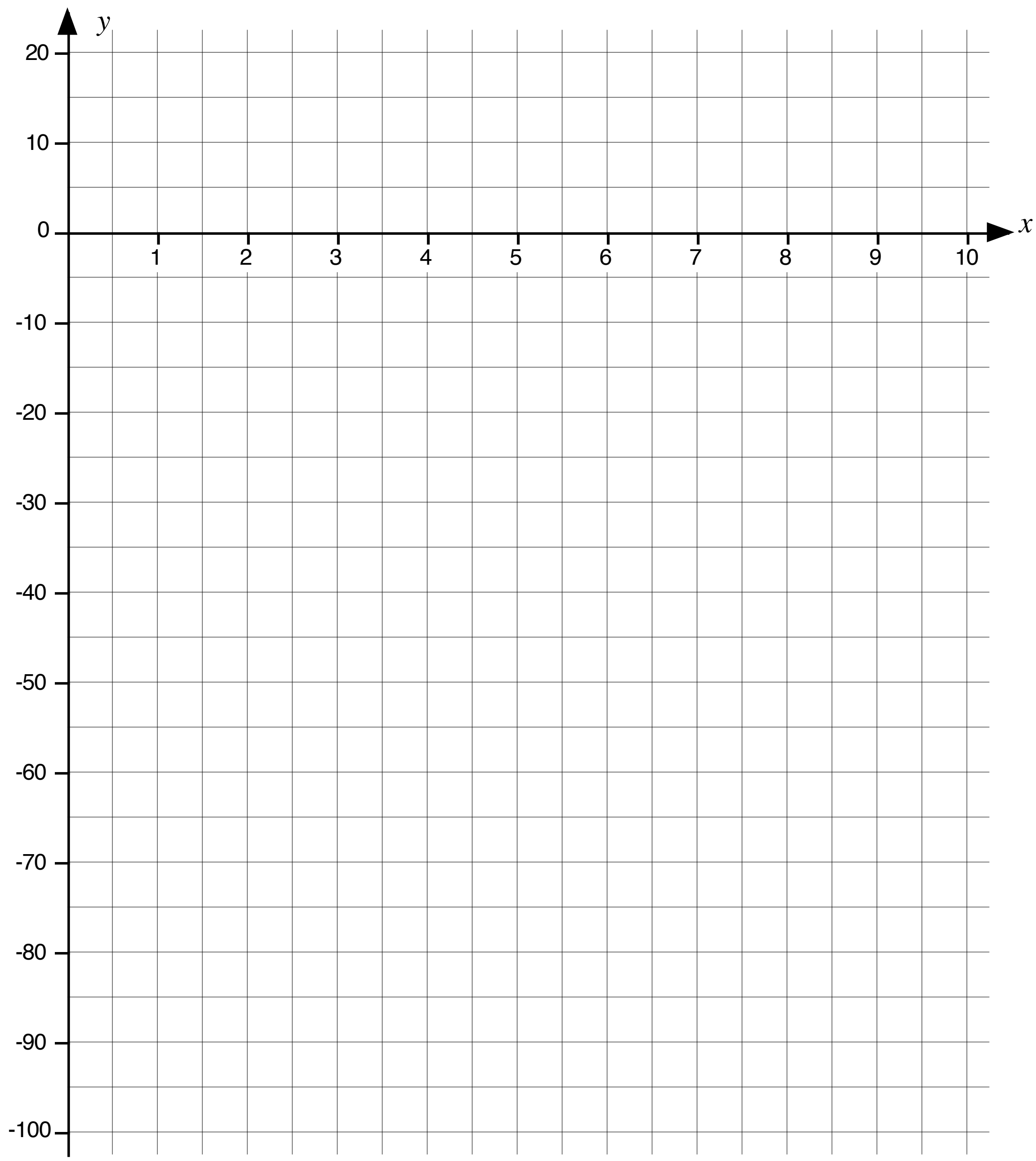
Sample Table 2:

Number of words remembered correctly at start of interval	Number of words remembered correctly at end of interval (x)	Number of words forgotten during interval	Length of time interval (days)	Rate of change (y)
10	7	3	1/24 – 0 = 1/24	(-1)*3 ÷ 1/24 = -72
7	5	2	1/3 – 1/24 = 7/24	(-1)*2 ÷ 7/24 = -48/7
5	4	1	1 – 1/3 = 2/3	(-1)*1 ÷ 2/3 = -3/2

Creating a Relationship

In this final part of the recitation you will create a relationship between the number of words remembered and the rate of change. This relationship will form the basis of the differential equation that you will create.

8. If you look carefully at Table 2 you will see that the entries in the second column are labeled as x and the entries in the last column are labeled y. Use the values from Table 2 as the x and y coordinates of points, and plot these points on the set of axes shown below.



When psychologists average the results of lots and lots of careful experiments together and create a graph like the one you have just made, they usually find that there is a linear relationship between x and y. (As our experiment was not very carefully controlled, and does not include very many data points, your graph might not look very much like a straight line.)

9. Use a ruler to draw the line that does the best job of getting as close to as many points on your graph as possible. Find the x and y coordinates of two points on this graph and the formula for the line that goes through them.

The meaning of the x that you used to create the graph is that x is the number of words you remember. The meaning of the y that you used to create the graph is that y is the rate of change of the number of words you remember. We could use the function $R(t)$ to represent the number of words you remember after t days and the derivative $\frac{dR}{dt}$ to represent the rate of change of the number of words you remember.

10. Write down the equation of the linear function you found. However, instead of y in this formula, write the derivative $\frac{dR}{dt}$ and instead of x in this formula, write R.

The equation that you have written down is a differential equation that defines the function $R(t)$, the number of words you remember after t days. In lecture on Wednesday we will figure out how to solve this equation to find a formula for the function $R(t)$.