Solutions for Recitation Handout 5

1. The completed table is shown below.

Year	1910	1911	1912	1913
Acres	139	150.12	162.1296	175.099968

2. You can use any of the entries to calculate the growth factor. We will use the first two.

$$B = \left(\frac{150.12}{139}\right)^{\frac{1}{1911-1910}} = 1.08.$$

3. The growth factor is equal to one plus the percentage growth expressed as a decimal (i.e. 8% is expressed as 0.08). There is a second formula for the growth factor, which is:

$$B = 1 + d$$

where d is the annual growth expressed in decimal form.

4. The growth factor will be $B = 1 + r/100$.

5. The initial size of the farm is 139 acres. This means that $A = 139$. The growth factor $B = 1.08$, so the formula for the size of the farm will be:

$$y = (139)\cdot(1.08)^x.$$

6. There are 86 years from 1910 to 1996. Assuming that the farm kept growing at a stead 8% for this entire period of time, gives that in 1996, the farm would cover:

$$y = (139)\cdot(1.08)^{86} = 104{,}101.6881 \text{ acres.}$$

7. If we let y represent the brown tree snake density (in units of snakes per square mile) and let x represent the number of years since 1952, then the snake density of Guam will be given by the exponential function:

$$y = (0.01)\cdot(1.28)^x.$$

To find out the snake density at the present time, we should plug $x = 2009 - 1952 = 57$ into this function. Doing that gives:

$$y = (0.01)\cdot(1.28)^{57} = 12{,}911.24939 \text{ snakes per square foot.}$$

The USGS estimate for 2009 is 13,000 snakes per square foot.

8. The three pieces of information contained in the U.S. State Department bulletin were:

- Officials described the situation in the former Soviet Union as **exponential growth** in the number of HIV/AIDS cases.
- In 2001, the U.S. Agency for International Development estimated that there were at least 103,000 HIV/AIDS cases in the Russian Federation.
- In 2002, the Associated Press reported that there were 250,000 HIV/AIDS cases in the Russian Federation.

This suggests that the function will be an exponential function that passes through the two points (11, 103000) and (12, 250000).

To find an equation for the function, first note that you are looking for an equation of the form:

$$y = A \cdot B^x$$

where A and B are constants. To determine the values of A and B you can follow the three steps that were introduced in class.

Step 1: Substitute the data points into the exponential equation

Using (12, 250000): $250000 = A \cdot B^{12}$

Using (11, 103000): $103000 = A \cdot B^{11}$

Step 2: Form a quotient of the equations and solve for *B*

$$\frac{250000}{103000} = \frac{A \cdot B^{12}}{A \cdot B^{11}} = B$$

Therefore, $B = 2.427184466$.

Step 3: Substitute the value for *B* and one of the data points into the exponential equation to solve for *A*

Using (11, 103000) and $B = 2.427184466$ gives:

$$A = \frac{103000}{(2.427184466)^{11}} = 5.980074591.$$

The completed equation for the function is:

$$y = (5.980074591) \cdot (2.427184466)^x.$$

9. To determine the number of HIV/AIDS patients in the Russian Federation in 2009, you can plug $x = 19$ into this formula. Doing this gives:

$$y = (5.980074591) \cdot (2.427184466)^{19} \approx 124{,}067{,}919.$$

10. To determine whether the relationship shown in Table 1 is perfectly exponential or not, you can calculate the growth factor using two different pairs of points. If you always get the same growth factor, then the data in the table show a perfectly exponential relationship.

Calculating the growth factor using (7, 6.12) and (8, 10.993)

$$B = \left(\frac{10.993}{6.12}\right)^{\frac{1}{8-7}} = 1.79624183.$$

Calculating the growth factor using (10, 89.796) and (11, 173)

$$B = \left(\frac{173.0}{89.796}\right)^{\frac{1}{11-10}} = 1.926589158.$$

The two growth factors are not identical, so the data in Table 1 does not show a perfectly exponential relationship. However, the growth factors are not wildly different, so the relationship between x and the number of HIV/AIDS cases in Table 1 can probably be approximated by an exponential function.

11. I suspect that the figure of about 124 million HIV/AIDS cases in the Russian Federation is probably too high. This is because the growth factors calculated using the values from Table 1 are both considerably lower than the growth factor of $B = 2.427184466$ used in Question 8. The lower growth factors would have led to a smaller estimate in Question 8. From a common sense point of view, the current population estimate for the Russian Federation (according to the CIA) is about 141 million and although HIV/AIDS is spreading rapidly in Russia, it is unlikely that six out of every seven Russians is HIV positive.