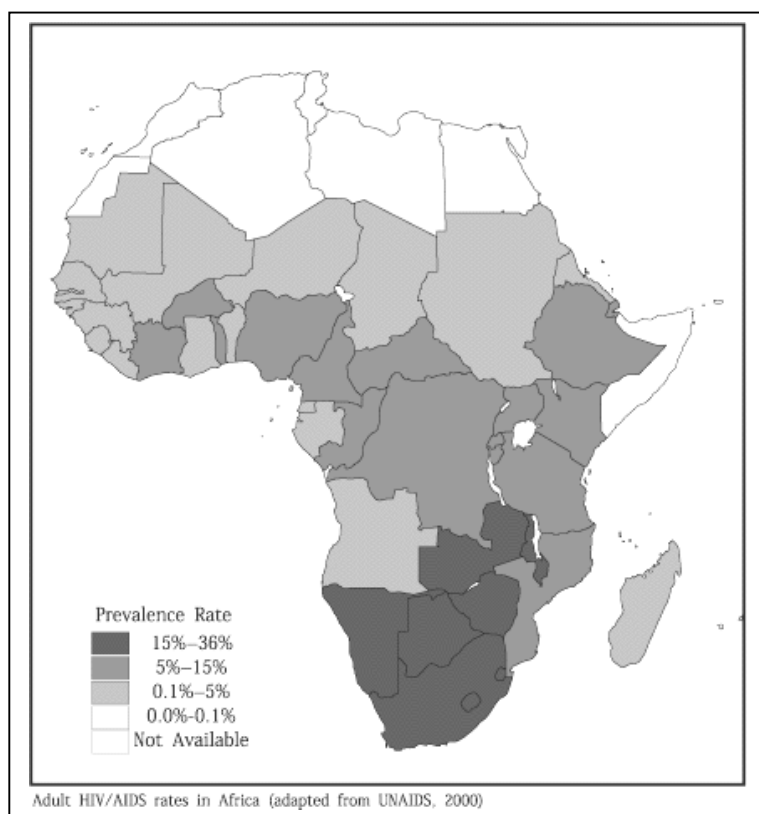


## Recitation Handout 2: Limits Involving Infinity

The specific learning goals of this activity are for you to:

- Use numerical and graphical representations of a function to search for evidence of a limit as the independent variable  $x \rightarrow \infty$ .
- Analyze the behavior of equations to determine whether or not a given function has a limit as the independent variable  $x \rightarrow \infty$ .
- Analyze the behavior of equations to calculate the numerical value of the limit of a given function as the independent variable  $x \rightarrow \infty$ .
- Learn about the scale of the impact that HIV/AIDS will have on the long-term growth of South Africa's population.



The region of the world that has been hardest hit by the HIV/AIDS pandemic is sub-Saharan Africa. In terms of the sheer numbers, South Africa is probably the country with the greatest number of people infected by HIV.

It is very difficult to appreciate the degree to which HIV/AIDS affects life in South Africa. Current estimates suggest that between 5 and 6 million<sup>1</sup> people (out of a total population of approximately 49 million<sup>2</sup>) are infected. The CIA estimates that among people aged 15-49, more than 20% of South Africa's people are HIV positive<sup>3</sup>. More than 50% of teenagers are expected to die before the age of 60, largely from HIV-related health problems<sup>4</sup>.

Half of the deaths in South Africa are now attributed to HIV/AIDS, and 71% of deaths among adults<sup>5</sup>. So many people have died from HIV-related health problems that some cemeteries<sup>6</sup> have run out of space, and South Africans now (on average) spend more time attending funerals than shopping<sup>7</sup>.

<sup>1</sup> Source: United Nations UNAIDS Report on the Global HIV/AIDS Epidemic, 2008.

<sup>2</sup> Source: CIA World Fact Book (estimate made in July, 2008).

<sup>3</sup> Source: CIA World Fact Book.

<sup>4</sup> Source: Center for Acturial Research, South African Medical Research Council.

<sup>5</sup> Source: Center for Acturial Research, South African Medical Research Council.

<sup>6</sup> Source: New York Times, July 29, 2004.

<sup>7</sup> Source: South Africa Advertising Research Foundation.



During the 1980's and 1990's, antiretroviral (ARV) drugs were developed that fought (but did not cure) HIV. When taken regularly these drugs helped to prevent or slow down the progression of HIV infection to AIDS. ARV medications could significantly extend the life and increase the quality of life for those who took them<sup>8</sup>. Two ARV drugs (called zidovudine and nevirapine) were also found to be effective in preventing mother-to-child (perinatal) transmission of HIV.

During the 1980's and 1990's, ARV drugs were very expensive (a one-year supply often costing \$15,000 or more) and were not available to most of the HIV positive people who lived in South Africa. However, during the 1990's, less expensive versions of the drugs were developed that were affordable for South Africans and the South African government.

Initially, American and European pharmaceutical companies opposed manufacture and distribution of cheap, effective ARV drugs with the argument that low-cost drugs could reduce their profits. (This argument was leant weight by former Presidents Clinton and George W. Bush.)

Despite opposition, low cost ARV drugs were developed and made available to the poorest nations of the world. Incredibly, the South African government (under the presidency of Thabo Mbeki) refused to distribute ARV drugs to HIV-positive South Africans, even when the drugs were offered at little or no cost<sup>9</sup>. For almost 10 years, Mr. Mbeki refused to accept the idea that HIV was the cause of AIDS, insisting instead that AIDS stemmed from poor nutrition or vitamin deficiencies.

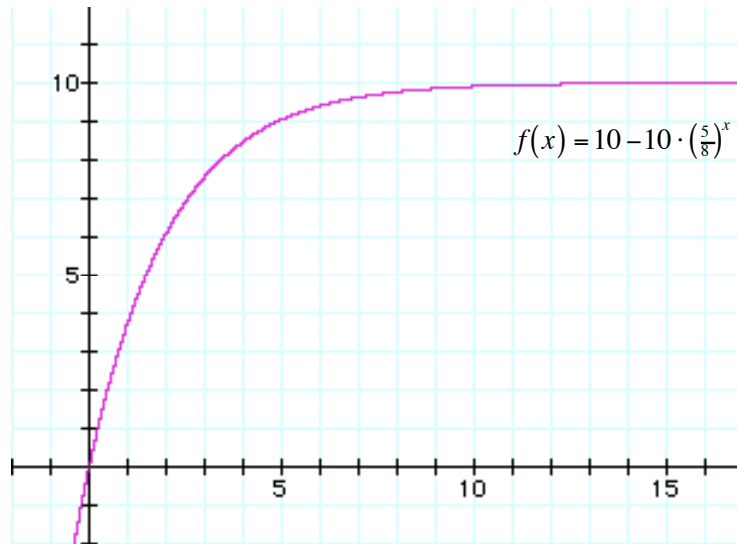
In 2001, an activist group (TAC, the Treatment Action Campaign) sued the South African government seeking an order to make nevirapine available to pregnant women who were HIV-positive. Although the South African constitutional court ruled for TAC, the government appealed the decision, dragging the legal battle out until government lawyers exhausted all of their appeals and the lawsuit finally concluded in 2008. At the time of writing, the South African government has started to distribute nevirapine in hospitals, clinics and health centers throughout the country although health ministry official continue to question the safety of the drug.

<sup>8</sup> The photograph included here shows the same man (who is HIV-positive) before (Photograph (a)) and after (Photograph (b)) beginning treatment with ARV medication. The source of this photograph is:

<http://www.impactaids.co.uk/>

<sup>9</sup> Source: Reuters, October 6, 2001. "Glaxo gives up rights to AIDS drugs in South Africa."

1. The graph and some numerical values of the function  $f(x) = 10 - 10 \cdot \left(\frac{5}{8}\right)^x$  are shown below. The function  $f(x) = 10 - 10 \cdot \left(\frac{5}{8}\right)^x$  approaches a limit of 10 as  $x \rightarrow \infty$ . In a few sentences, briefly explain which features of the graph and table (given below) suggest that this might be the case.



$x$	0	5	10	100	1000	10000
$f(x)$	0	9.046	9.909	$\approx 10$	$\approx 10$	$\approx 10$

2. The South African government is still faced with the decision of whether or not to distribute antiretroviral drugs that will significantly reduce the chances for HIV to develop into full-blown AIDS. Two models of population growth have been developed<sup>10</sup> to describe how the South African population will change if the government does decide to distribute antiretroviral drugs and to routinely use drugs such as zidovudine<sup>11</sup> to reduce the incidence of perinatal transmission of HIV from mother to child. The two models that have been developed are:

**Exponential:**  $P = (37.819) \cdot (1.02056)^x$ .

**Logistic:**  $P = \frac{89.808}{1 + (1.48) \cdot (0.9552)^x}$ .

where the independent variable,  $x$ , is number of years since 1990 and the dependent variable,  $P$ , is the population of South Africa (in units of millions of people).

Use the axes on the next page to sketch accurate graphs of these two functions and determine whether either function is likely to have a finite limit as  $x \rightarrow \infty$ .

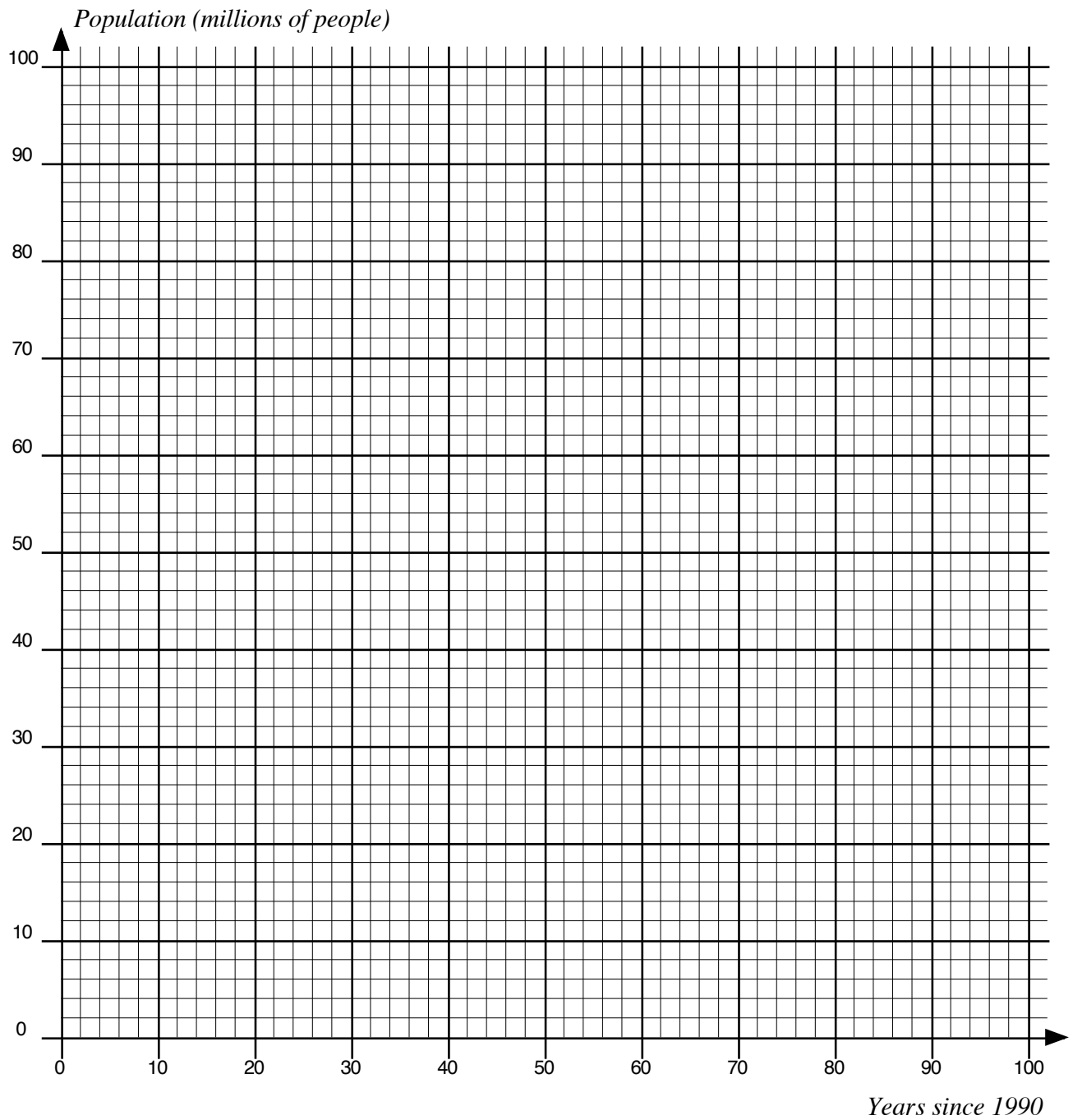
3. Confirm your speculations from Question 2 by using the algebraic structure of each function to explain why or why it does not have a finite limit as  $x \rightarrow \infty$ . As part of your answer, you should obtain a precise numerical value for the finite limit (if either function actually has one).

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<sup>10</sup> The data used to construct these models was obtained from the following sources:

- ING Barings. 1999. *The Demographic Impact of AIDS on the South African Economy*. Report available from: ING, 2 Merchant Place, Fredman Drive, Sandton 2196, West Crescent, Johannesburg, South Africa.
- ING Barings. 2000. *Economic Impact of AIDS in South Africa: A Dark Cloud on the Horizon*. Report available from: ING, 2 Merchant Place, Fredman Drive, Sandton 2196, West Crescent, Johannesburg, South Africa.
- Arndt, C. and J. D. Lewis. 2000. The macro implications of HIV/AIDS in South Africa: A preliminary assessment. Paper presented at the IAEN Symposium on "The Economics of HIV/AIDS in Developing Countries." (Durban, South Africa, July 7-8 2000.)

<sup>11</sup> See: Mofenson, L. M. 1999. Can perinatal HIV infection be eliminated in the United States? *Journal of the American Medical Association*, 272: 577-579.



**KEY**

<input type="checkbox"/>	HIV/AIDS treatment provided (Exponential growth)	<input type="checkbox"/>	No HIV/AIDS treatment provided (Logistic growth)
<input type="checkbox"/>	HIV/AIDS treatment provided (Logistic growth)		

4. South Africa's current political leadership appears to be reluctant to distribute HIV treatment drugs to its citizens<sup>12</sup>, regularly falling behind distribution schedules set by the Ministry of Health. A population growth model has also been developed<sup>13</sup> to describe how the South African population will grow if the current government policies and trends in the spread of HIV/AIDS continue. This model is described by the logistic growth equation given below.

**Logistic growth:** 
$$P = \frac{50.349}{1 + (0.702) \cdot (0.8196)^x}.$$

Where the independent variable,  $x$ , is the number of years since 1990 and the dependent variable,  $P$ , is population of South Africa (in units of millions of people).

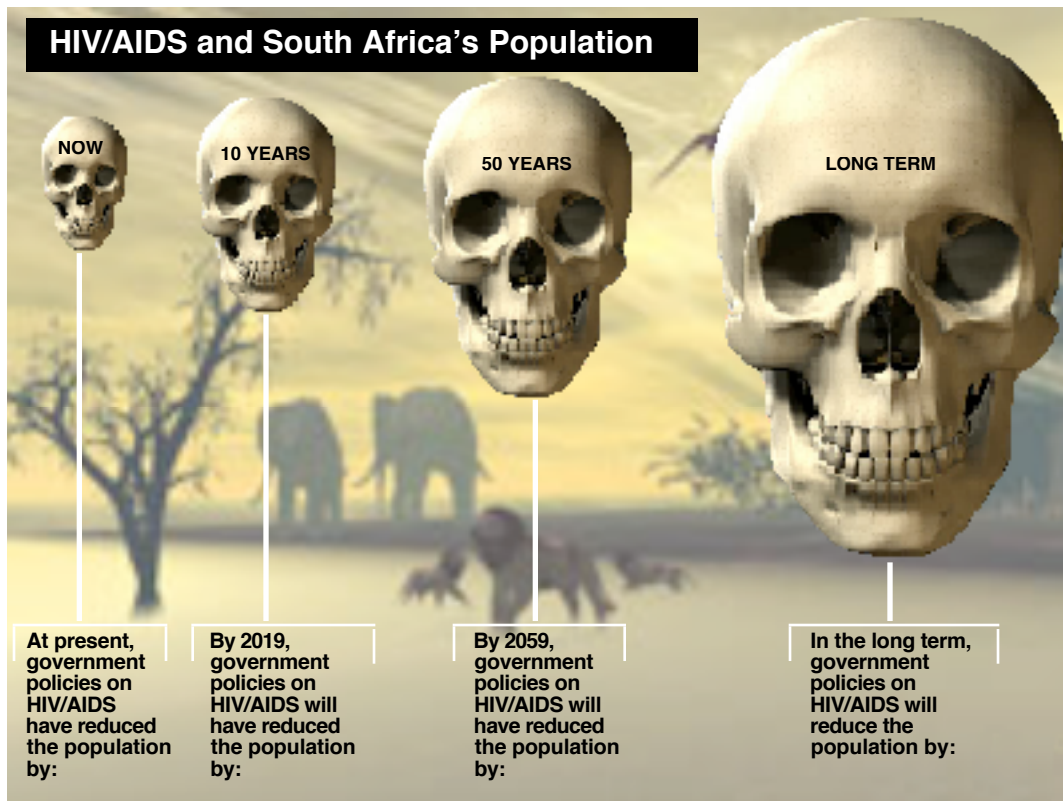
Sketch an accurate graph of the population of South Africa assuming that the government continues with its current policies regarding HIV/AIDS medications. Does this function have a finite limit as  $x \rightarrow \infty$ ? If so, briefly explain how you could deduce this from the equation for population growth, and find the precise numerical value of the limit as  $x \rightarrow \infty$ .

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<sup>12</sup> For a more recent report on the South African government's spending policies, see: Carroll, Roy. 2002. Anger at Mbeki's 'vulgar' £33m jet. *The Guardian*, October 24 2002. This story is available on-line at: <http://www.guardian.co.uk/international/story/0,3604,817790,00.html>

<sup>13</sup> The data sources for this model are identical to those for the models given in Question 2.

5. Of the two functions proposed in Question 2, most experts believe that the logistic growth equation will provide the most accurate description of South Africa's population growth. In the long term (say, in the next 50 or 100 years) how much of a difference will South African government policies on HIV/AIDS medications have on the size of the South African population? Express your answer by completing the graphic<sup>14</sup> shown below.



## Epilogue

Some examples of genocide that have been directly related to supposedly non-genocidal government policies implemented during times of peace include:

- **Ukraine, 1932-1933:** 4,821,600 starve to death in the former Soviet Union's most productive agricultural region as Joseph Stalin and his henchman Lazar Kaganovich attempt to force Ukrainian peasants to abandon their centuries-old family-held farming operations and form large agricultural collectives.
- **Cambodia, 1975-1979:** 1,700,000 are tortured, executed, starved or killed by lack of access to basic medical treatment as Pol Pot and the Khmer Rouge government force Cambodians to abandon cities and return to the supposed Utopia of an agrarian society.
- **East Timor, 1975-1999:** 200,000 civilians are killed when East Timor declares independence from Portugal and is subsequently invaded (and occupied for over 20 years in defiance of U.N. Security Council resolutions) by the Indonesian Security Forces.
- **Rwanda, 1993-1994:** 700,000 (predominantly Tutsi) are killed when a small group radical Hutus organize a countrywide program of slaughter. Rwanda is a former colony of Germany, which was administered by Belgium when Germany was stripped of its colonial possessions after World War 1. Both German and Belgian governments turned the traditional Tutsi-Hutu tribal relations into a class system, severely favoring the Tutsi minority. In return for high status, privilege and education, the Tutsi became enforcers of European rule.

<sup>14</sup> This graphic was created using images from: <http://www.ecran3d.com/webhtm/web27/savana.jpg> and <http://gw.marketingden.com/planets/images/subrender/Skull.html>