Solutions for Water Security Handout

(a) Very roughly speaking (to do this calculation exactly, you have to use exponential functions an logarithms, which you will learn about later in the semester) if the people of Botswana are using 4% of their water each year, then they will have consumed all (i.e. 100%) of their water in:

$$\frac{100}{4} = 25 \text{ years.}$$

This means that Botswana will have exhausted their water supply by the year 2028.

(b) The graph is shown as Figure 3 (see next page). Note that the lines that are drawn on the graph are part of the answer to Part (d).

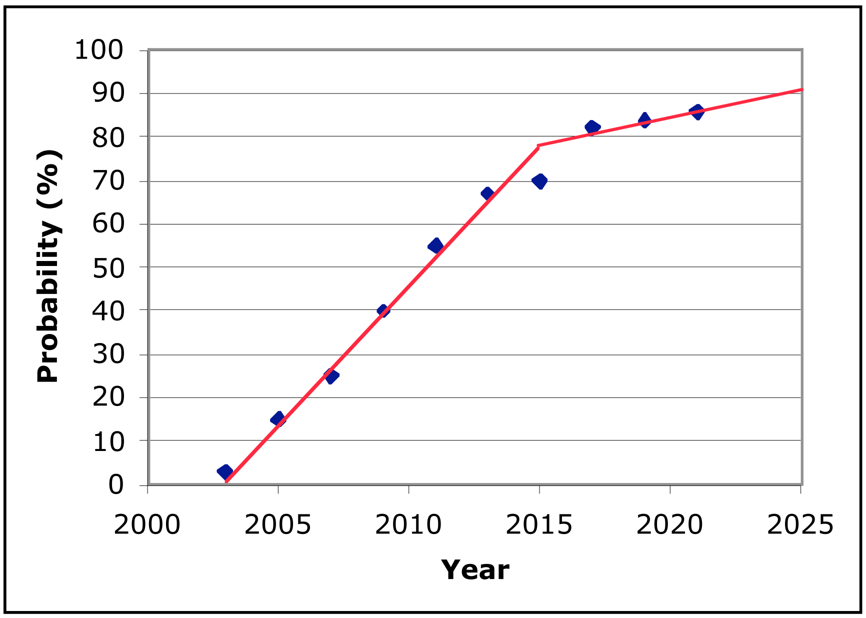

Figure 3: Relationship between year and probability that Botswana will exhaust its water reserves.

(c) Currently (2005) a substantial percentage of the population of Botswana is HIV-positive, and the virus is spreading rapidly through the population. It takes about 10 years for a person infected with HIV to develop AIDS and then die. Given this incubation period of ten years, I would expect to see the effects of HIV and AIDS become dramatic in the years following 2005 + 10 = 2015. If you look at Figure 3, you can see that around the year 2015, the STATPLOT flattens out suddenly. This may be due to the fact that many people will be dying of AIDS in Botswana from the year 2015 onwards, reducing the size of the population and reducing the amount of water used each day.

(d) Two linear functions that you could use to represent the relationship are shown in Figure 3 (above).

(e) The first, steep line goes through the two points (2005, 15) and (2011, 55). Calculating the slope of the line that goes through these two points gives:

$$y = 6.67 \cdot x - 13358.35.$$

The second, shallower line goes through the two points (2019, 84) and (2021, 86). Calculating the slope of the lines that goes through these two points gives:

$$y = x - 1935.$$

The "change over" from one the first formula to the second formula occurs at about x = 2015. This means that the first linear function will be valid when x is between 2003 and 2015, and the second linear function will be valid when x is greater than 2015. There is nothing in this problem to suggest which of the two linear functions should apply when you are actually at the point x = 2015 so you could choose either. In this solution I am making the (arbitrary) choice to make the first linear function (and not the second) valid at the point x = 2015.

Representing all of this in "function defined in pieces" notation gives the following. Note the "≤" between the x and the 2015 to show that the first linear function is valid at the point x = 2015.

$$y = \begin{cases} 6.67 \cdot x - 13358.35 & , \quad 2003 \le x \le 2015 \\ x - 1935 & , \quad 2015 < x \end{cases}.$$

(f) The second of the two linear functions will be the one that applies when the probability gets close to 100%. You can determine the year when the probability reaches 100% by plugging y = 100 into the second linear equation, and then solving for x.

$100 = x - 1935$ (Plug y = 100 into the equation)

$x = 2035$ (Add 1935 to both sides of the equation)

According to the linear functions found in Part (e), the probability that Botswana's water reserves will be exhausted reaches 100% in the year 2035.

(g) The modification that is needed is that after x = 2035, the output value of the function should be fixed at 100 (see Figure 4, below). The modified function defined in pieces if given below.

$$y = \begin{cases} 6.67 \cdot x - 13358.35 & , \quad 2003 \le x \le 2015 \\ x - 1935 & , \quad 2015 < x \le 2035 \\ 100 & , \quad 2035 < x \end{cases}$$

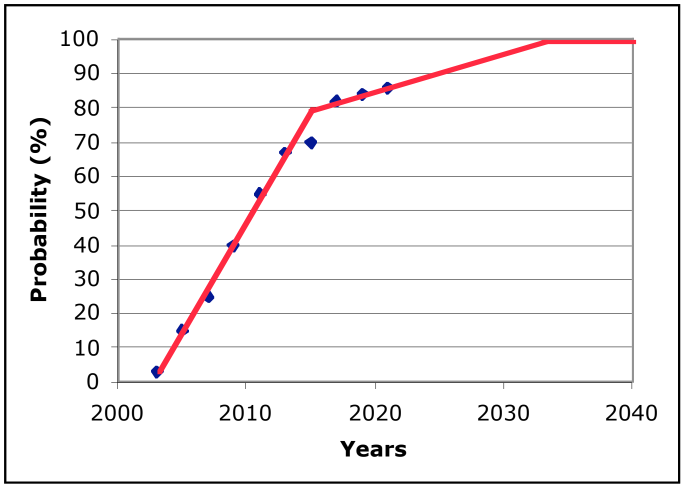

Figure 4: Relationship between year and probability that Botswana will exhaust its water reserves.