Recitation Handout 10: Experiments in Calculus-Based Kinetics

Today's recitation will focus on curve sketching. These are problems where you information about the first and second derivative of a function and you job is to reconstruct the graph of the function itself.

However, instead of just drawing a curve to represent your interpretation of the first and second derivative, you will have to go one step further. In addition to understanding the information from the first and second derivative well enough to be able to visualize the original function graph that the derivatives came from, you will have to devise a way of moving that corresponds to your interpretation of the derivatives.

The specific goals of today's recitation are for you to:

1. Learn how the relationship between distance, time and velocity can be expressed using functions and derivatives.
2. Practice the skill of sketching the graph of a function, given the graph of its derivative.
3. Practice the skill of using information about the first and second derivative of a function to sketch the graph of a function.

Distance, Velocity and Acceleration

As many of you will already know, distance, velocity and acceleration are quantities that are related by derivatives.

- ***Velocity*** is the first derivative of *distance* with respect to *time*. If the function $s(t)$ gives distance after t seconds have elapsed, then velocity is given by the derivative $s'(t)$.
- ***Acceleration*** is the first derivative of *velocity* with respect to *time*. This also makes acceleration the second derivative of distance with respect to time. If the function $s(t)$ gives distance after t seconds have elapsed, then acceleration is given by the derivative $s''(t)$.

Familiarizing Yourself with the Equipment

In this recitation you will be working with the experimental apparatus shown in Figure 1[1].

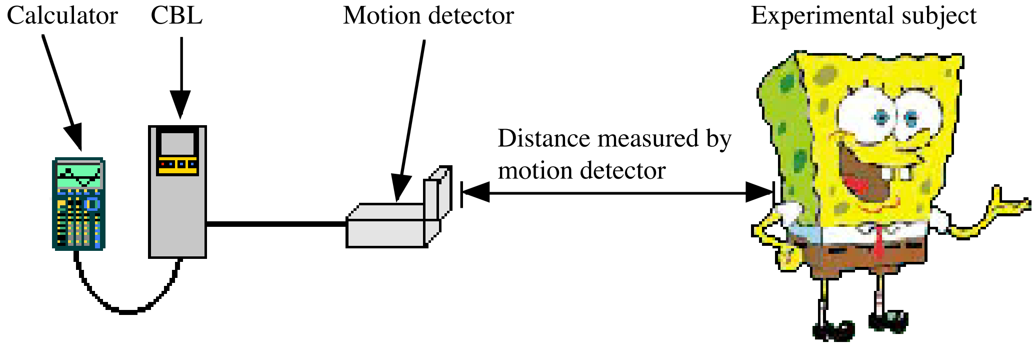

Figure 1: Experimental equipment and set up.

Your recitation instructor will operate the technology. Your role in the recitation will be the one played by SpongeBob in Figure 1, namely the person running back and forth whose motion produces a graph on the calculator screen.

In this context, when you move **away** from the motion detector, your **distance will increase** and your **velocity will be positive**.

On the other hand, when you move **towards** the motion detector, your **distance will decrease** and your **velocity will be negative**.

The **concavity** of your graph will be related to your **acceleration**. When you accelerate (it doesn't matter whether you go towards or away from the motion detector) the graph you produce should have a **concave up** shape. When you **decelerate**, the graph that appears on the calculator screen should have a **concave down** shape.

When you are reading your graphs and trying to figure them out, always remember that **graphs are read from left to right**.

A Practice "Run"

The next page shows a graph. This graph is a graph of distance from the motion detector (vertical axis) plotted against time (horizontal axis).

1. With the other people in your class, decide how a person should move in order to produce a graph like the one shown on the next page. When you think you know how to do this, have your recitation instructor operate the technology and try out your idea. If the graph that is produced does not resemble the graph that you were given, rethink your ideas and try again.

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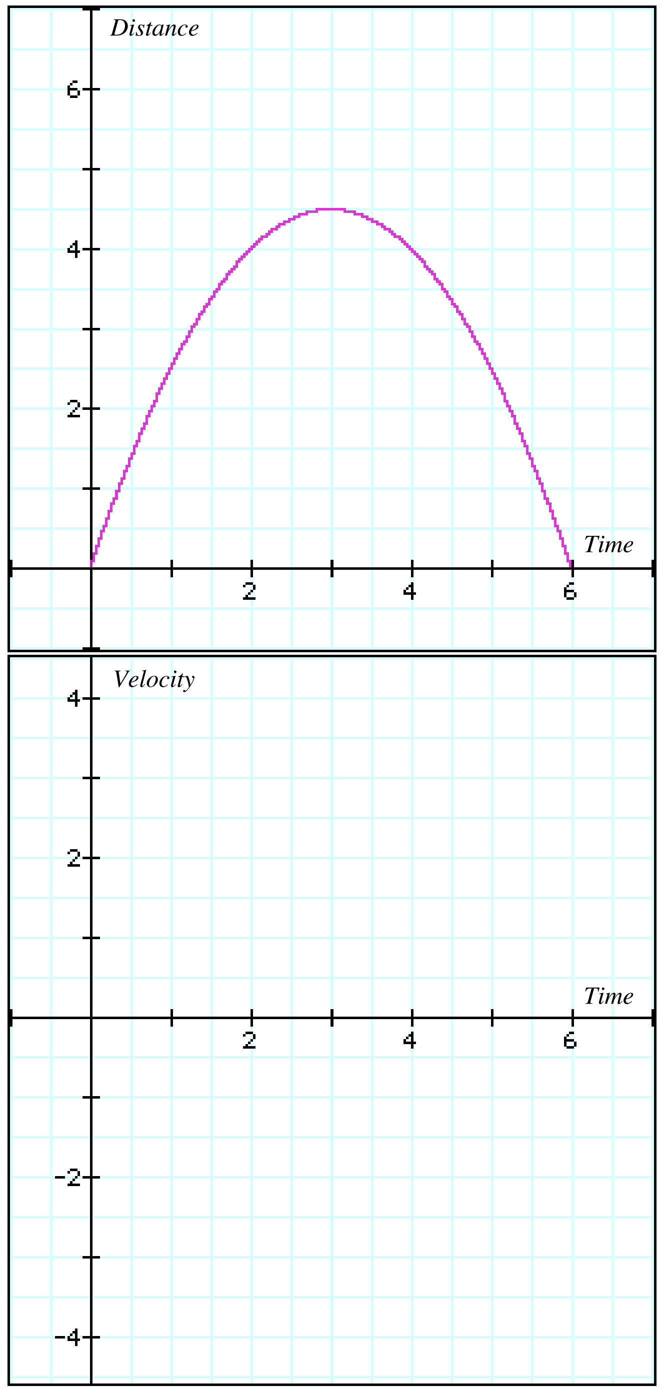
[1] Figure 1 was created using images from http://aleandrarose.tripod.com/ and http://www.ti.com/

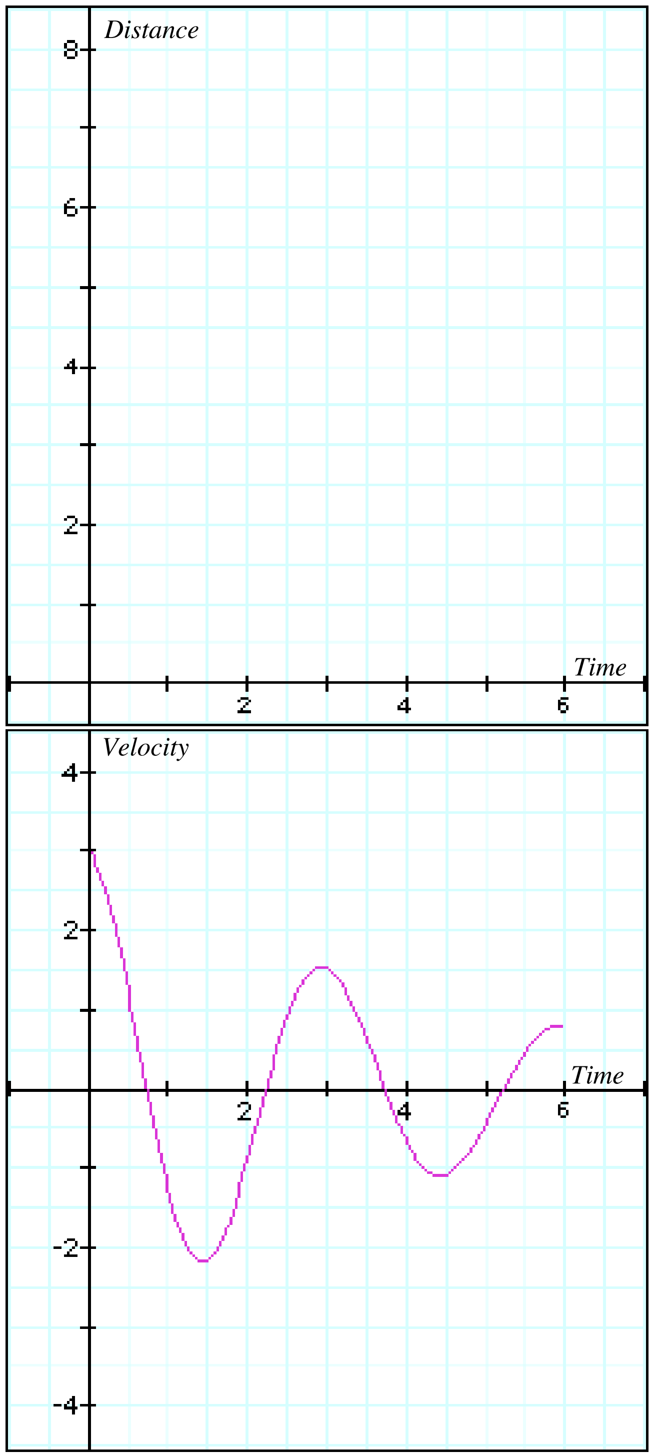


2. Use the axes provided above to sketch the graph of velocity versus time.

- How can you deduce the velocity (approximately) at each point from the graph given above?
- How can you tell when the person should turn around from the graph given above?
- According to the graph that you were given, should the person be accelerating or decelerating throughout the run? How can you reconcile with what the person actually has to do to produce the graph?

Working Backwards from Velocity

3. The graph given below shows the *velocity* of a runner in front of the motion detector. Use this graph to draw a plausible *distance* graph for the runner. Remember that the distance between the runner and the motion detector cannot go below zero.



4. Once you have your graph, compare it to the graphs drawn by the other people in your recitation and iron out any discrepancies that you observe.

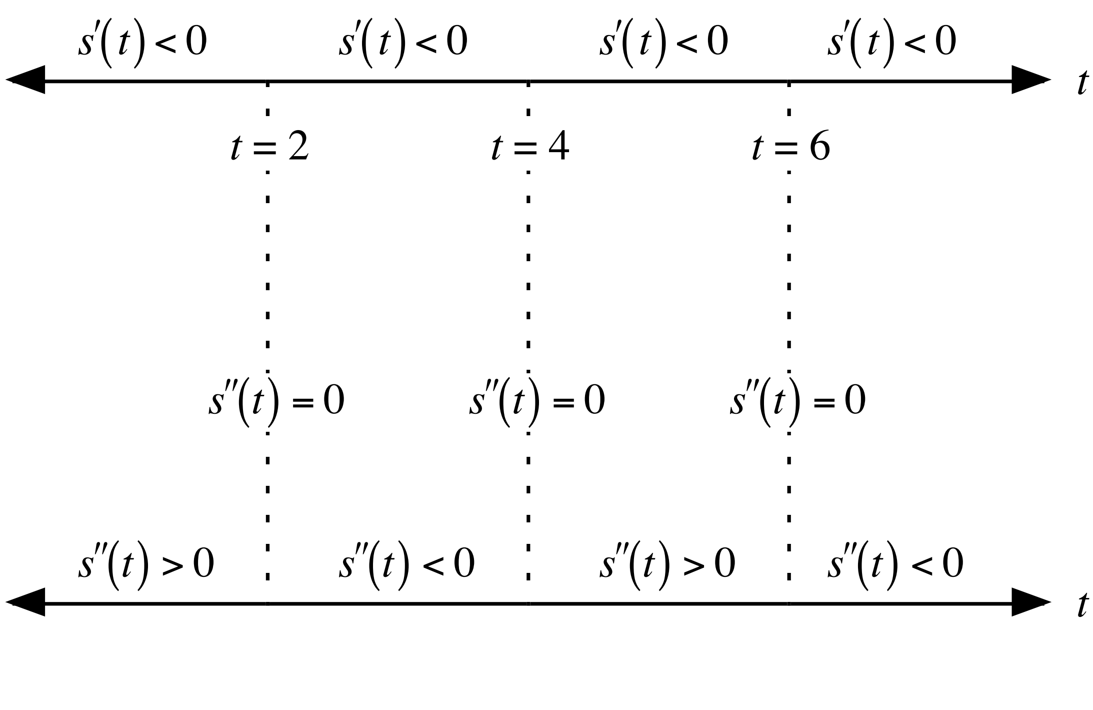
5. Once you and the people you are working with manage to produce a distance graph that you can all believe in, discuss what a person would actually have to do in order to produce such a graph by running back and forth in front of the motion detector.

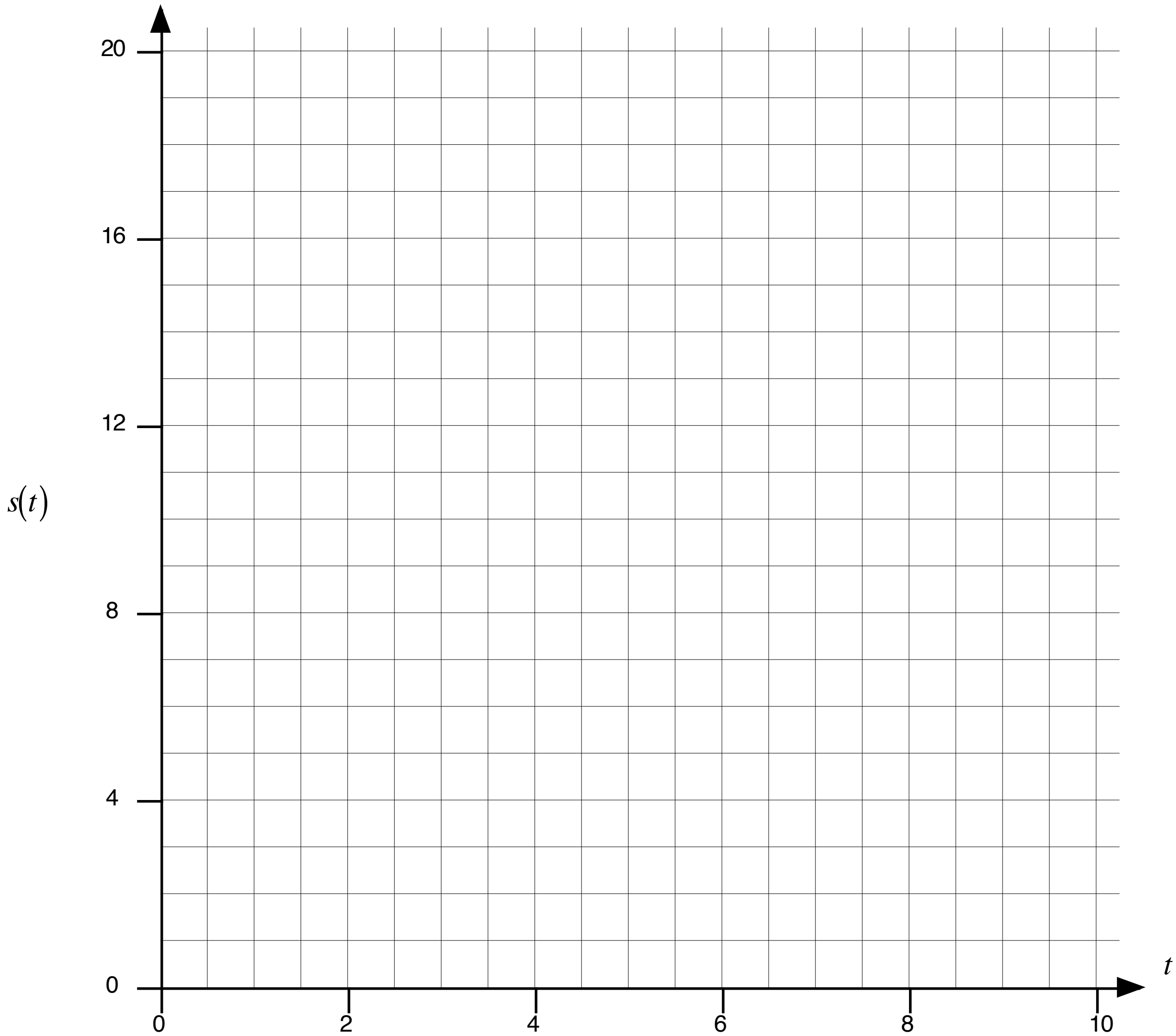
6. Once you and the people you are working with agree on a plan, try it out by actually running in front of the motion detector to see if you can produce a graph that looks like the *distance* graph that you have drawn. If the graph does not come out right, rethink your plan and try again.

Working Directly from Derivatives

Suppose that the function $s(t)$ gives the distance of a runner from the motion detector t seconds after it has been activated.

7. You are told that $s(0) = 20$ and given the number lines shown below. Use this information and the axes provided on the next page to draw a plausible *distance* graph.





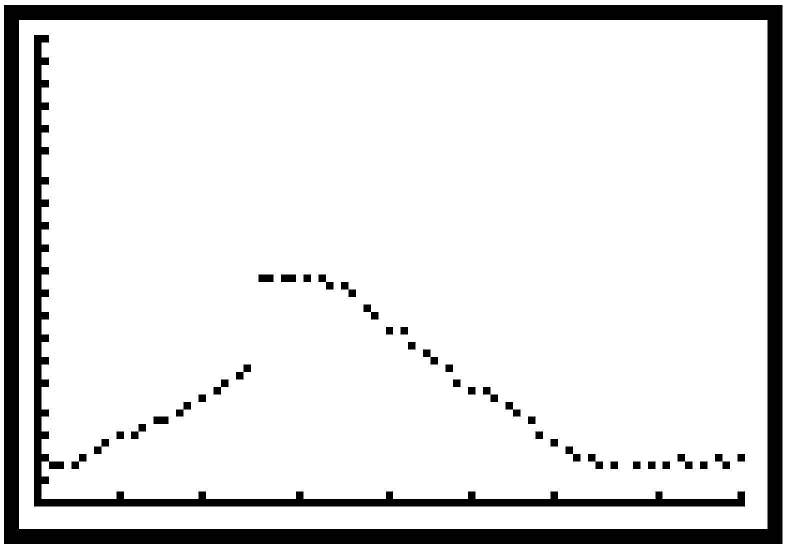
8. Once you have your graph, compare it to the graphs drawn by the other people in your recitation and iron out any discrepancies that you observe.

9. Once you and the people you are working with manage to produce a distance graph that you can all believe in, discuss what a person would actually have to do in order to produce such a graph by running back and forth in front of the motion detector.

10. Once you and the people you are working with agree on a plan, try it out by actually running in front of the motion detector to see if you can produce a graph that looks like the *distance* graph that you have drawn. If the graph does not come out right, rethink your plan and try again.

A Challenge for the Imagination

11. The graph given below shows a **real** distance-time graph that was produced by students working with exactly the same equipment you have been working with today. Try to figure out what they did and come up with a plan that would allow you to produce a similar graph for yourself.



12. Once you and the people you are working with agree on a plan, try it out by actually running in front of the motion detector to see if you can produce a graph that looks like the *distance* graph that you have drawn. If the graph does not come out right, rethink your plan and try again.