## Recitation Handout 1: Water Security in Sub-Saharan Africa

The specific learning goals of this activity are for you to:

- Use data to create formulas for linear functions.
- Create a formula for a function defined in pieces.
- Understand how real world events (such as the HIV/AIDS epidemic) are impacting public policy in African countries.
- Predict when the African nation of Botswana will use up all of its fresh water reserves.
- Modify the formula of a function to take "real world" considerations into account.


Figure 1: The southern African country of Botswana and the location of the capital city of Gaborone.

At the moment, the country with the fastest spread of HIV and AIDS is the southern African nation of Botswana[1] (see Figure 1[2]). The CIA estimates that 330,000 of the country's 1,573,267 people are HIV-positive, almost 21% of the total population. Among people aged 15-49, the percentage is 38.8%[3].

Botswana was ruled by Great Britain until gaining independence in 1966. Botswana is home to some of the most spectacular scenery and animal life in the world (see Figure 2[4]). Although tourism is an important component of the economy, the country's main industry is diamond mining, which generates 30-35% of the country's gross domestic product (GDP)[5]. Botswana produces approximately 50% of the world's diamonds.

The Botswana HIV epidemic is poised to severely reduce the size of Botswana's population over the course of the next twenty-five years. HIV is not the only serious problem that confronts the people of Botswana, however.

In addition to HIV, Botswana faces problems of environmental degradation due to overgrazing of savannah by domesticated cattle, droughts and crop failures that are further exacerbated by changing global climate conditions, and severely limited access to fresh water.

Researchers at the International Institute for Applied Systems Analysis[6] (IIASA), a non-governmental organization (NGO) based in Austria, estimate that the inhabitants of Botswana's capital city of Gaborone consume more than 4% of Botswana's total fresh water reserves every year.

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[1] Source: CNN. *Zimbabwe AIDS epidemic shows no sign of letting up*. March 12, 1999.
[2] Image source: http://www.hivafrica.org/
[3] Source: CIA World Fact Book, 2005 Edition. Available on-line from: http://www.cia.gov/
[4] Image source: http://www.discover-botswana.com/
[5] Source: CIA World Fact Book, 2005 Edition. Available on-line from: http://www.cia.gov/
[6] For more about this organization, see: http://www.iiasa.ac.at/docs/IIASA_Info.html

Figure 2: Botswana is home to some of the world's most spectacular and interesting wildlife.

**(a)** The human inhabitants of Botswana currently consume more than 4% of the country's fresh water supplies every year. Estimate the length of time that the people of Botswana have before their reserves of fresh water are totally exhausted.

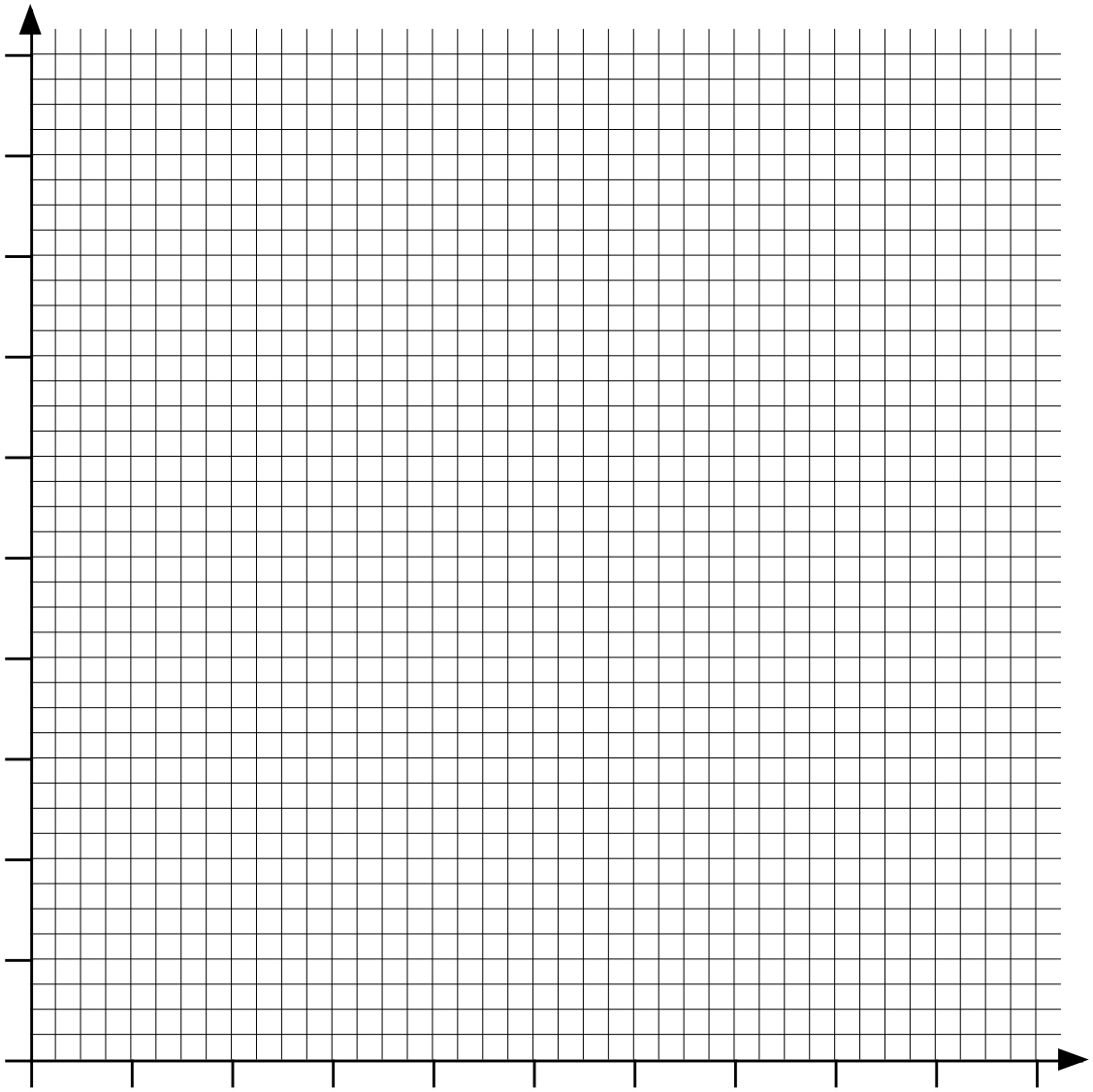
Dividing 100 by 4 and estimated that Botswana's water reserves will be completely exhausted by the year 2028 is a pretty good approximation. Given the seriousness of the problem (i.e. no water whatsoever for drinking, hygiene, industry, agriculture, etc.) the government of Botswana is interested in obtaining the most accurate figures possible. In 1996 a group of researchers created a complicated mathematical model for water use in Botswana and used this to calculate the probability that Botswana would run out of water in any particular year. The results[7] that the researchers obtained are shown in Table 1 (below).

| Year | 2003 | 2005 | 2007 | 2009 | 2011 | 2013 | 2015 | 2017 | 2019 | 2021 |
|---|---|---|---|---|---|---|---|---|---|---|
| Probability (%) | 3 | 15 | 25 | 40 | 55 | 67 | 70 | 82 | 84 | 86 |

Table 1: Probability that Botswana will completely exhaust its fresh water.

**(b)** Using the year as the input and the probability as the output, use the axes provided on the next page to create a graph to show the relationship between years and the probability that Botswana will exhaust its supplies of fresh water.

[7] Source: Strzepek, K. M., M. Hellmuth, A. Holt, N. Mladenov. 1998. Water resources system modeling: An important piece in the population-development-environment puzzle. *Origins*. 8(3): 38-45.



**(c)** Botswana is currently in the grip of a major HIV epidemic, with the fastest rate of HIV transmission of any country in the world. HIV, in and of itself, is not lethal for humans. However, the presence of HIV in a person's system may lead to the development of Acquired Immune Deficiency Syndrome (or AIDS) that compromises the immune system. When a person's immune system is compromised, they can die of a disease (such as toxoplasmosis or pneumonia) that would not normally be lethal. It takes about 10 years for most people to develop AIDS once they have contracted HIV. Based on the numbers given in Table 1 and the appearance of your graph from Question (b), will the HIV/AIDS epidemic have any effect on the probability that Botswana will exhaust its water supplies? How do you know?

**(d)** The relationship between year and percentage can be represented fairly well using a piecewise function built up out of two linear functions. On the graph that you drew in Part (b), sketch these two linear functions.

**(e)** Find the formulas of the two linear functions that you drew in Part (d), and use these to write down a "function defined in pieces" that gives the probability that Botswana will exhaust its water supplies.

**(f)** Based on the equations that you found in Part (e), when will the probability reach 100%? That is, in what year will it certain that Botswana will have exhausted all of its water reserves?

**(g)** The function that you set up as a function defined in pieces is supposed to give the probability that Botswana will run out of water. The highest that a probability can get is 100%. Modify the function that you set up in Part (e) to reflect this.