Handout 21: The technique of u-Substitution

Find an equation for each of the anti-derivatives (or indefinite integrals) given in the table below. In each case, identify the "inside function" (u), calculate the derivative $\frac{du}{dx}$ and rewrite the indefinite integral in terms of u and du.

Anti-derivative (Indefinite integral)	***Work and equation for Anti-derivative***
$\int \ln(x) \cdot \sqrt{x \cdot \ln(x) - x} \cdot dx$	
$\int \left(\frac{-1}{x^2} + 2x\right) \cdot \sqrt{\frac{1}{x} + x^2 + 1} \cdot dx$	

Answers: (a) $(2/3)*[x*\ln(x) - x]^{3/2} + C$. **(b)** $(2/3)*[1/x + x^2 + 1]^{3/2} + C$.
(c) $2*[x*\ln(x)]^{1/2} + C$. **(d)** $(1/15)*[x + \ln(x)]^{15} + C$. **(e)** $\ln(\ln(x)) + C$.

Anti-derivative (Indefinite integral)	***Work and equation for Anti-derivative***
$\int \left(\ln(x)+1\right)\cdot \frac{1}{\sqrt{x\cdot \ln(x)}}\cdot dx$	
$\int \left(\frac{x+1}{x}\right)\cdot \left(x+\ln(x)\right)^{14}\cdot dx$	
$\int \frac{1}{\ln(x)}\cdot \frac{1}{x}\cdot dx$	

Answers: **(a)** $(2/3)*[x*\ln(x) - x]^{3/2} + C$. **(b)** $(2/3)*[1/x + x^2 + 1]^{3/2} + C$.
(c) $2*[x*\ln(x)]^{1/2} + C$. **(d)** $(1/15)*[x + \ln(x)]^{15} + C$. **(e)** $\ln(\ln(x)) + C$.