Handout 15: What, exactly, is a logarithm?

Logarithms

Logarithms are algebraic tools that enable you to solve exponential equations in an algebraic way. This saves you the (sometimes considerable) trouble of thinking up a viewing window that will enable you to solve the exponential equation using the INTERSECT capability of a graphing calculator.

Example

> "Now the seven angels holding the seven trumpets prepared to blow them. The first angel blew his trumpet, and there was hail and fire mixed with blood, and it was thrown down at the Earth so that a third of the Earth was burned up, a third of the trees were burned up, and all the green grass was burned up. Then the second angel blew his trumpet, and something like a great mountain of burning fire was thrown into the sea. A third of the sea became blood, and a third of the creatures living in the sea died, and a third of the ships were completely destroyed. Then the third angel blew his trumpet, and a huge star burning like a torch fell from the sky, it landed on a third of the rivers and on the springs of water. (Now the name of the star is Wormwood.) So a third of the waters became wormwood, and many people died from these waters because they had turned bitter."
>
> – REVELATION (8: 6-12).

> "Chornobyl (also spelled Chernobyl or Czarnobyl in Russian) is a little-known Ukrainian word for the medicinal herb *Artemisia*, commonly called wormwood."
>
> – PROCEDINGS OF THE INTERNATIONAL ATOMIC ENERGY ASSOCIATION CONFERENCE HELD ON THE 10TH ANNIVERSARY OF THE CHERNOBYL ACCIDENT (VIENNA, AUSTRIA, APRIL 8-12, 1996).

On April 26 1986, the world's worst nuclear accident occurred in the #4 reactor at the Chernobyl power plant in the Ukraine. The accident spread biologically dangerous radioactive isotopes throughout the northern hemisphere. Potentially dangerous levels of fallout were detected in Washington State, New York City, Idaho, New Jersey, Alaska, Hawaii, Vermont, Wyoming, Nebraska, Virginia, Georgia, Tennessee, Maine and Oregon. Some of the radiation was detected in the ground, some in the groundwater, some in the air, and some in milk[1].

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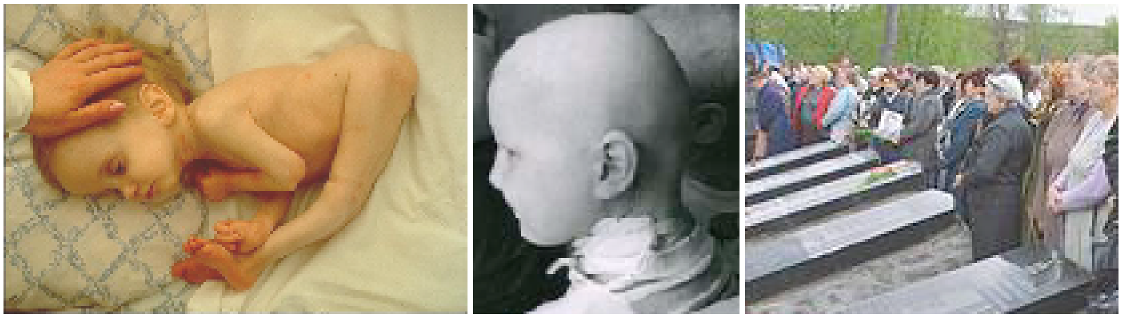
[1] These statements are based on measurements reported in the published scientific studies listed below. Except where otherwise noted, these studies are from: "Environmental Measurements Laboratory: A Compendium of the Environmental Measurements Laboratory's Research Projects Related to the Chernobyl Nuclear Accident." Report Number EML-460, Department of Energy, New York, 1986.



The legacy of the 1986 Chernobyl accident. Deformity. (This poor child has relatively minor birth defects by current Ukrainian and Belarussian standards.) Disease and cancer. (This poor girl wears a bandage around her neck, over the surgical wound made during her thyroid cancer treatment. Such bandages are now so common that they are nicknamed "Belarussian necklaces.") Death.

While disturbing, the levels of radiation detected in the United States were dwarfed by environmental readings in the Ukraine and Belarus[2], and by the levels of radioactivity that continue to be recorded there today. In a series of events eerily reminiscent of Revelation 8:12, heavy rains following the accident forced many radioactive isotopes deep into the groundwater systems of Ukrainian and Belarussian territories. It is a well-substantiated fact that the incidence of cancer – especially thyroid cancer among young people – has increased dramatically in both the Ukraine and Belarus in the wake of the Chernobyl accident (see below – the information for France is included for comparative

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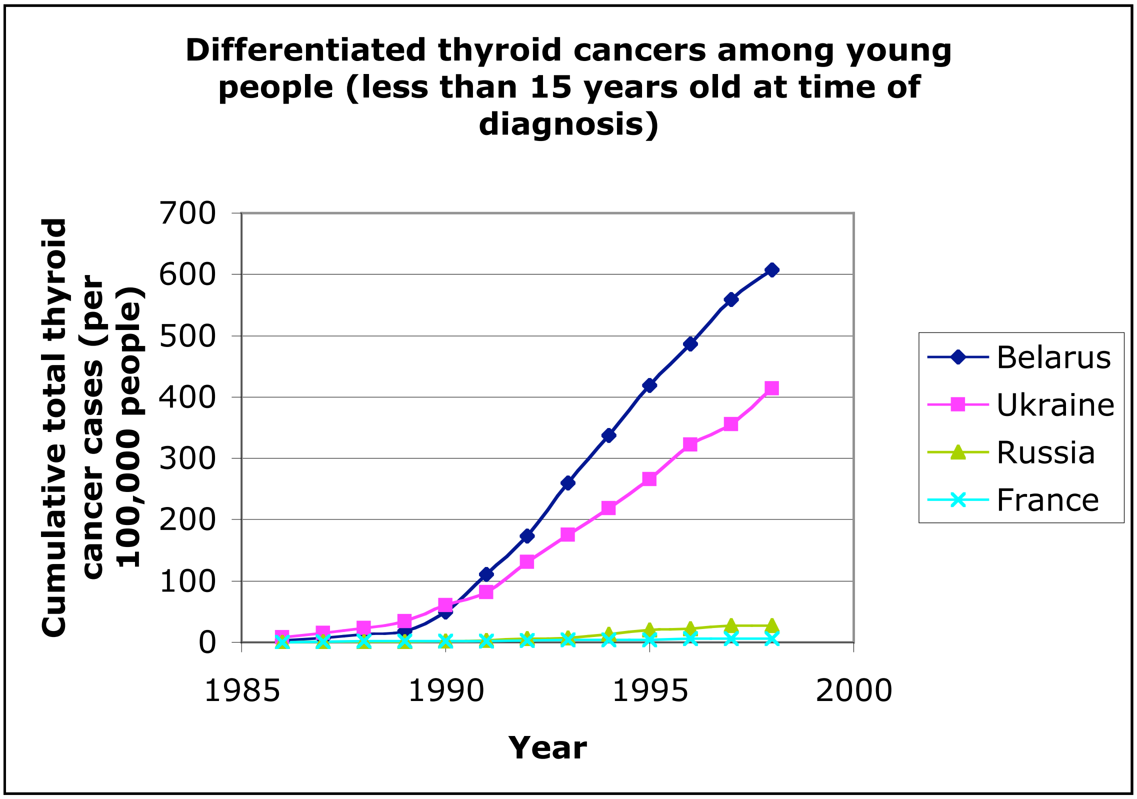
• Z. J. Juzdan, I. K. Helfer, K. M. Miller, W. Rivera, C. G. Sanderson and S. Silvestri. "Decomposition of radionuclides in the northern hemisphere following the Chernobyl accident."
• R. J. Larsen, C. G. Sanderson, W. Rivera, M. Zamichieli. "The characterizations of radionuclides in North American and Hawaiian surface air and deposition following the Chernobyl Accident."
• R. J. Larsen, P. L. Haagenson and N. M. Reiss. "Transport processes associated with the initial elevated concentrations of Chernobyl radioactivity in surface air in the United States." *Journal of Environmental Radioactivity*, **10**: 1-18, 1989.
• United States Environmental Protection Agency. "Environmental radiation data: Report 46. April 1986-June 1986." Report Number EPA 520/5-87-004. Environmental Protection Agency, Washington, DC, 1986.
• J. E. Dibb and D. L. Rice. "Chernobyl fallout in the Chesapeake Bay region." *Journal of Environmental Radiation*, **7**: 193-196, 1988.
• M. Dreicer and C. S. Klusek. "Transport of I-131 through the grass-cow-milk pathway at a northeast US dairy following the Chernobyl accident." *Journal of Environmental Radioactivity*, **7**: 201-207, 1988.
• M. Dreicer, I. K. Helfer and K. M. Miller. "Measurement of the Chernobyl fallout activity in grass and soil at Chester, New Jersey."
• H. W. Feely, I. K. Helfer, Z. R. Juzdan, C. S. Klusek, R. J. Larsen, R. Leifer and C. G. Sanderson. "Fallout in the New York metropolitan area following the Chernobyl accident." *Journal of Environmental Radioactivity*, **7**: 177-191, 1988.
• C. S. Klusek, C. G. Sanderson and W. Rivera. "Concentrations of I-131, Cs-134 and Cs-137 in milk in the New York metropolitan area following the Chernobyl reactor accident."
• K. M. Gebbie and R. D. Paris. "Chernobyl: Oregon's response." Radiation Control Section, Office of Environment and Health Systems, Health Division, Oregon Department of Human Resources. Portland, OR, 1986.
• E. A. Bondietti and J. N. Brantley. "Characteristics of Chernobyl radiation in Tennessee." *Nature*, **322**: 313-314, 1986.

[2] Image sources: http://cems.alfred.edu/ http://www.adiccp.org/ http://news.bbc.co.uk/

purposes). Truly it may be the case that "…many people died from these waters because they had turned bitter."



(Source of data: Aurengo, A. 2002. Chernobyl: The effects on public health. *Forum on Physics and Society*. 31(4): 11-26.)

The "accident" in reactor number 4 was not an accident in the purest sense – human error and poor judgment made significant contributions to the disaster.

Reactor 4 was scheduled to be shutdown for routine maintenance on April 25, 1986. Somebody in the power plant – although precisely who has never been determined – decided to take advantage of the shutdown to run a test. The object of this test was to determine whether, in the event of a loss of electrical power, the power plant's steam turbine could continue to produce enough electrical power to operate the emergency equipment and coolant pumps until the emergency diesel generator came on-line.

For the test, the reactor was to be stabilized at a power output of 1000MW. However, an operator error allowed the power output to fall to 30MW. In order to correct for this, reactor personnel withdrew all but 6 of the control rods used to control the nuclear reaction. Standard operating procedure at the power plant required that a minimum of 30 control rods be inserted into the atomic pile at any time. This meant that if a power surge occurred – with an accompanying surge in radioactivity – at least 20 seconds would be needed to insert enough control rods into the atomic pile to stop the nuclear reaction. At precisely this moment, a power surge occurred and after only a few seconds, the atomic pile exploded.

Figure 1: Sattelite photograph of the Chernobyl power station, April 30, 1986. The light areas in the middle are the actual fires. The dark area in the middle is smoke.

The highest levels of radiation from Chernobyl were detected in northern European countries[3], Scandinavian countries[4] and the Baltic States[5].

The Chernobyl accident occurred at the height of the Cold War. As such, Soviet officials may have been reluctant to announce the Chernobyl accident to the world, despite the fact that the accident had potentially global consequences. Swedish nuclear technicians detected the first signs of trouble at the Forsmark Nuclear Power Plant 60 miles north of Stockholm[6]. The Swedes detected abnormally high levels of radiation at approximately 9am on April 28 (2 days after the meltdown) and thought that their own reactor was leaking. After a frantic search revealed nothing wrong, the Swedes checked the prevailing wind patterns and questioned the Soviet government. Finally, at 9pm that night (after a full day of silence despite direct, desperate inquiries from Western nations), the newscaster on the official Soviet television station read the following statement (reproduced *in full* and *verbatim* here):

> "An accident has taken place at the Chernobyl power station, and one of the reactors was damaged. Measures are being taken to eliminate the consequences of the accident. Those affected by it are being given assistance. A government commission has been set up."

After that terse statement, the newscaster went on to discuss another news item.

Tens - possibly hundreds - of thousands[7] of deaths in the former Soviet Union and Europe have been connected to radiation from the Chernobyl meltdown. Scientists[8] have

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[3] For example, see: K. Iriweck, B. Khademi, E. Henrich and R. Kronraff. "Pu-239(240), Sr-90, Ru-103 and Cs-137 concentrations in surface air over Austria due to dispersion of Chernobyl releases over Europe." *Journal of Environmental Radioactivity*, **20**: 133-148, 1993.

[4] For example, see: A. Aarkrog. "Studies of Chernobyl debris in Denmark." *Environment International*, **14**: 149-155, 1988.

[5] For example, see: E. Realo, J. Jogi, R. Koch and K. Realo. "Studies on radiocesium in Estonian soils." *Journal of Environmental Radioactivity*, **29**: 111-120, 1995.

[6] Source: John Greenwald. "Deadly meltdown." *TIME Magazine*, May 12, 1986.

[7] The United Nations Office for the Coordination of Human Affairs reports that 7.1 million people in the Ukraine and neighboring countries have required special care for cancer and radiation sickness. The International Federation of the Red Cross has reported that cancer rates in these areas are 16 times higher than normal. European Union scientists estimate that more than 20,000 premature deaths in Europe may be directly attributed to Chernobyl radiation.

estimated that worldwide, as many as 2.9 billion people may have been exposed to biologically dangerous radioactive materials ejected from the Chernobyl accident.

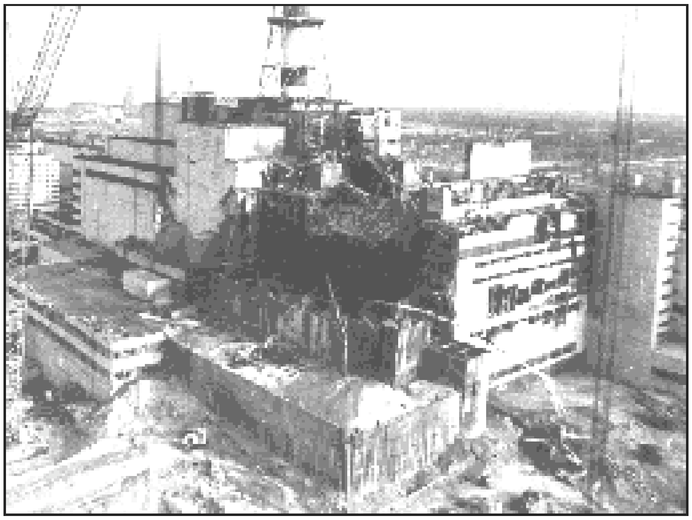

Figure 2: The number 4 reactor unit at Chernobyl. This photograph was taken when the first in the number 4 reactor had finally been supressed. Note the streams of water directed onto the reactor from fire trucks in the hope of keeping the reactor core from re-igniting.

Astonishingly, the Chernobyl plant continued to operate for another 14 years until it was finally deactivated on December 15, 2000[9]. In 1986, workers encased the dangerously radioactive Number 4 reactor in a "Sarcophagus" of steel, lead, soil and concrete (see Figure 3). With the problem supposedly contained, the remaining three reactors in the power plant continued to operate.

According to the estimates of some European scientists[10], significant quantities of radioactive fallout were released into the atmosphere for as many as ten days following the accident.

"Fallout" consists of radioactive particle that can enter the body by breathing, or through food and drink. Radioactive fallout is carried by the wind and the amount of fallout that a person is exposed to decreases the further they are from the nuclear blast.

Table 1[11] gives the radiation dose from radioactive fallout that a person would be subjected to if they were caught in the open and unprotected when the fallout from a nuclear explosion occurred.

In this example x will represent the miles from the center of the nuclear blast and y will represent the dose of radiation in units of rads.

x	6.25	12.5	18.75	25.0	31.25	37.5	43.5	50.0	56.25	62.5
y	1402	1056	822	654	542	448	364	300	252	205

Table 1: Radiation dose for an unprotected person caught in the open.

[8] See: L. R. Anspaugh, R. J. Catlin and M. Goldman. "The global impact of the Chernobyl reactor accident." *Nature*, **242**: 1516, 1988.
[9] Source: http://www.newsmax.com
[10] See: Sich, A. R. 1996. Truth was an early casualty. *Bulletin of the Association of Atomic Scientists*. 52(3): 4-7.
[11] Source: NATO Handbook on the Medical Aspects of NBC Defensive Operations. Part I: Nuclear. *NATO Publication APMed6-(B).*

(a) Is the relationship between x and y perfectly exponential? Is it approximately exponential?

(b) Use regression on a graphing calculator to find the formula for the function that best approximates the relationship between x and y.

(c) In February 2002, the Nuclear Regulatory Commission forced the closure of the Davis-Besse nuclear power located in Oak Harbor, OH. This distance between Oak Harbor and Bowling Green is about 46 miles. What dose of radiation would you receive if there was an accident at the Davis-Besse reactor and you were outside when the radioactive fallout arrived? For the sake of comparison, about 25% of adults who receive a radiation dose of 300 rads will die within 30 days of exposure.

(d) In medical terminology "LD-50" refers to the dose at which 50% of adults who receive the dose will die. For the radioactive fallout created by a catastrophic nuclear reactor accident, the LD-50 is 450 rads. How close would you have to be to the Davis-Besse plant to have a 50% chance of dying in the event of a serious accident?

Solution

(a) To determine whether or not the data in Table 1 is perfectly linear or not, you can calculate the growth factor using different pairs of points. If you always get exactly the same value for the growth factor then the data is perfectly exponential. If you get roughly the same growth factor each time then the data is not perfectly exponential but can probably be quite well approximated using an exponential function.

Calculating the Growth Factor Using (6.25, 1402) and (12.5, 1056):

$$B = \left(\frac{1056}{1402}\right)^{\frac{1}{12.5-6.25}} = 0.955666901.$$

Calculating the Growth Factor Using (18.75, 822) and (25.0, 654):

$$B = \left(\frac{654}{822}\right)^{\frac{1}{25-18.75}} = 0.9640797236.$$

This reveals that the data in Table 1 is not perfectly exponential. However, the two growth factors are quite close, so an exponential function will probably do a pretty good job of representing the relationship between x and y.

(b) Regression on a calculator gives that the exponential function that represents the relationship between x and y is:

$$y = (1587.0133183843) \cdot (0.96724451141363)^x.$$

(c) To determine the dose that you would receive if you were outside when the fallout arrived in Bowling Green, you can plug $x = 46$ into the formula from Part (b). Doing this gives:

$$y = (1587.0133183843) \cdot (0.96724451141363)^{46} = 342.97 \text{ rads}.$$

This is more than sufficient to kill 25% of adults who are exposed to it.

(d) To work this out you need to determine the value of x that will give a y-value of 450. That is, you need to solve the exponential equation:

$$450 = (1587.0133183843) \cdot (0.96724451141363)^x$$

for x. Using logarithms to do this:

$$\frac{450}{1587.0133183843} = (0.96724451141363)^x \qquad \text{(Divide both sides by 1587)}$$

$$\log\left(\frac{450}{1587.0133183843}\right) = \log\left((0.96724451141363)^x\right) \qquad \text{(Take logs)}$$

$$\log\left(\frac{450}{1587.0133183843}\right) = x \cdot \log(0.96724451141363) \qquad \text{(Bring the } x \text{ down)}$$

$$\frac{\log\left(\frac{450}{1587.0133183843}\right)}{\log(0.96724451141363)} = x \qquad \text{(Divide by log(0.9672))}$$

$$x = 37.844 \qquad \text{(Evaluate on a calculator)}$$

So, if you were within 37 or 38 miles of the power plant when a catastrophic accident occurred, you would receive a dose of 450 rads and have a 50% chance of dying from radiation poisoning within 30 days of the exposure.

Example

Figure 3: Located near Harrisburg, PA, the Three Mile Island nuclear power station was the site of America's worst nuclear accident. Beginning at about 4am on Wednesday March 28, 1978, a combination of equipment failures and human errors resulted in the release of approximately 250,000 gallons of radioactive water from the plant and almost caused a melt-down of the reactor core.

The worst civilian[12] nuclear accident in U.S. history occurred on March 28, 1978 at the Three Mile Island nuclear power station (see Figure 3[13]).

When a serious accident or dangerous condition develops in a nuclear reactor the normal course of action is to SCRAM the reactor, which shuts down the nuclear reaction very quickly and if done in time, will prevent a nuclear explosion.

The word "SCRAM" is an acronym that stands for Safety Cut Rope Axe Man. This expression was adopted by the nuclear pioneer and Nobel laureate Enrico Fermi. When Fermi built his first reactor, the SCRAM mechanism consisted of a big block of cadmium metal that was suspended above the reactor by a thick rope. Fermi paid an expert lumberjack to stand beside the nuclear reactor – axe at the ready – and chop through the rope when the reactor started to get out of control and a nuclear explosion was imminent.

SCRAMming a nuclear reactor is a serious undertaking, especially for a nuclear reactor that generates electrical power. After a SCRAM, it is difficult, expensive and time-consuming to restart the reactor. What is more, while the reactor is shut down, it is not generating electricity and so is not making money for the power company that operates it. As you can imagine, nuclear technicians will only SCRAM a commercial reactor when a serious disaster is imminent.

It is easy to dismiss disasters such as Chernobyl and Three Mile Island as isolated incidents that resulted either from poor reactor design or operator error. In this example you will look at the average number of times each nuclear power plant in the U.S. in SCRAMmed each year – that is, how many times a potential disaster develops in every U.S. nuclear power plant each year.

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[12] In addition to above ground atomic tests conducted in Nevada in the 1950's, the U.S. military has experienced a number of serious accidents with nuclear reactors. Most notably, in 1955 and 1961 accidents occurred in the experimental SL-1 reactor operated by the U.S. Army near Idaho Falls, Idaho. During the 1961 accident the reactor accidentally achieved a supercritical condition and exploded killing three army personnel outright and contaminating a large area of Idaho.

[13] Image source: http://www.epa.gov/ If you are interested in learning more about the Three Mile Island accident, you can find a thorough description at: http://www.pbs.org/wgbh/amex/three/

In this example, x will always represent the number of years since 1980 and y will always represent the average number of SCRAMs for each nuclear power plant in the United States each year.

Table 2[14] gives the average number of SCRAMs each year for each individual nuclear power plant in the U.S. between 1988 and 2000.

Year	x	SCRAMs
1988	8	2.27
1989	9	1.98
1990	10	1.74
1991	11	1.59
1992	12	1.35
1993	13	1.19
1994	14	1.04
1995	15	0.92
1996	16	0.77
1997	17	0.68
1998	18	0.60
1999	19	0.53
2000	20	0.46

Table 2: Average numbers of SCRAMs for each nuclear power plant.

(a) Is the data in Table 2 perfectly exponential or not? If not, could the data in Table 2 be approximated using an exponential function?

(b) Find the formula for the function that best represents the relationship between x and y.

(c) The Three Mile Island accident occurred in 1978 ($x = -2$). According to the formula that you found in Part (b), what was the average number of SCRAMs in each nuclear plant when the Three Mile Island accident occurred?

(d) According to the formula that you found in Part (b), in what year were SCRAMs a monthly occurrence? (That is, in what year did each nuclear power plant in the U.S. have an average of 12 SCRAMs per year?)

(e) According to the formulas that you found in Part (b), the nuclear power industry is getting safer and safer as time goes by. In what year will the average number of SCRAMs reach 0.01? (This means that the reactors will have become so safe that SCRAMs occur – on average – only once a century.)

[14] Source: Nuclear Regulatory Commission. http://www.nrc.gov/

Solution

(a) To determine whether the data in Table 2 is perfectly exponential or not, you can calculate the growth factor using several different pairs of points. If the growth factor always comes out exactly the same then the data is perfectly exponential. If the growth factor is always roughly the same, then the data is not perfectly exponential but will probably be well approximated by an exponential function.

Calculating the growth factor using (8, 2.27) and (9, 1.98)

$$B = \left(\frac{1.98}{2.27}\right)^{\frac{1}{9-8}} = 0.872246696.$$

Calculating the growth factor using (10, 1.74) and (11, 1.59)

$$B = \left(\frac{1.59}{1.74}\right)^{\frac{1}{11-10}} = 0.9137931034.$$

These growth factors are not identical, so the data in Table 2 is not perfectly exponential. However, there is not that much difference between then, so an exponential function will probably do a reasonable job of representing the relationship between x and y.

(b) If you enter the data from Table 2 into a calculator and perform an exponential regression then you can get the function:

$$y = 6.704520631 \cdot (0.8747484981)^{x}.$$

(c) To determine the average number of SCRAMs in 1978, you can plug $x = -2$ into the formula from Part (b). Doing this gives:

$$y = 6.704520631 \cdot (0.8747484981)^{-2} = 8.76 \text{ SCRAMs in each nuclear power plant.}$$

(d) To answer this question, you have to find out what the value of x is when $y = 12$. That is, you have to solve the exponential equation:

$$12 = 6.704520631 \cdot (0.8747484981)^{x}$$

for x. Doing this with logarithms:

$$\frac{12}{6.704520631} = (0.8747484981)^{x} \qquad \text{(Divide by 6.7045)}$$

$$\log\left(\frac{12}{6.704520631}\right) = \log\left((0.8747484981)^x\right) \qquad \text{(Take logs)}$$

$$\log\left(\frac{12}{6.704520631}\right) = x \cdot \log(0.8747484981) \qquad \text{(Simplify)}$$

$$\frac{\log\left(\frac{12}{6.704520631}\right)}{\log(0.8747484981)} = x \qquad \text{(Divide by log(0.8747))}$$

$$x = -4.35 \qquad \text{(Evaluate on calculator)}$$

This would correspond to the year 1975 or 1976.

(e) To answer this question, you have to find out what the value of x is when y = 0.01. That is, you have to solve the exponential equation:

$$0.01 = 6.704520631 \cdot (0.8747484981)^x$$

for x. Doing this with logarithms:

$$\frac{0.01}{6.704520631} = (0.8747484981)^x \qquad \text{(Divide by 6.7045)}$$

$$\log\left(\frac{0.01}{6.704520631}\right) = \log\left((0.8747484981)^x\right) \qquad \text{(Take logs)}$$

$$\log\left(\frac{0.01}{6.704520631}\right) = x \cdot \log(0.8747484981) \qquad \text{(Simplify)}$$

$$\frac{\log\left(\frac{0.01}{6.704520631}\right)}{\log(0.8747484981)} = x \qquad \text{(Divide by log(0.8747))}$$

$$x = 48.63 \qquad \text{(Evaluate on calculator)}$$

This would correspond to the year 2028 or 2029.

The Concept of an Inverse

The mathematical relationship that reverses the roles of the dependent and independent variable is called an **inverse relationship**. If a function uses x as its input and gives y as its output, then the **inverse relationship** takes y as the input and gives x as the output.

For example, if you read about the Uighurs and the Desert mummies in the notes on exponential functions, then you will have set up a function that takes the age of a desert mummy as the input and gives the amount of carbon-14 that remains in the mummy's tissues as the output.

$$\begin{tikzcd}
\text{INPUT} & T \arrow[d] & \text{AGE IN YEARS} \\
\text{FUNCTION} & U \arrow[d] & \\
\text{OUTPUT} & U(T) & \text{AMOUNT OF C-14}
\end{tikzcd}$$

But what we were really interested in doing was starting with an amount of carbon-14 and determining the age of one of the desert mummies. To do this, we had to work back the other way – starting with the amount of carbon-14 that was present and going back to the age in years.

$$\begin{tikzcd}
& T \arrow[d] & \text{AGE IN YEARS} & T = U^{-1}(c) & \\
\text{FUNCTION} & U \arrow[d] & & U^{-1} \arrow[u] & \text{INVERSE} \\
& c = U(T) & \text{AMOUNT OF C-14} & c \arrow[u] &
\end{tikzcd}$$

The relationship that "undoes" or "reverses" the function is called the **inverse** of the function.

If $f(x)$ is a function and $g(x)$ is the inverse, then when you combine $f(x)$ and $g(x)$ by **composition**, the resulting composite function just gives x as the output.

$$f(g(x)) = x$$

$$g(f(x)) = x.$$

The inverse of the function $f(x)$ is often written using the symbols: $f^{-1}(x)$.

Finding the formula for the Inverse of a Function

When you have a formula for a function $f(x)$, you can sometimes find a formula for the inverse of the function.

Example

Find the equation for the inverse of:

$$y = f(x) = \frac{x}{x+1}$$

Solution

Step 1: Swap the x's and the y's

$$x = \frac{y}{y+1}.$$

Step 2: Rearrange to make y the subject of the equation

$x \cdot (y+1) = y$	(Multiply both sides by $(y + 1)$)
$x \cdot y + x = y$	(Distribute the x across the $(y + 1)$)
$x = y - x \cdot y$	(Get everything involving y on the same side of the equation)
$x = y \cdot (1 - x)$	(Factor out the y)
$\frac{x}{1-x} = y$	(Divide by $(1 - x)$)

The notation $f^{-1}(x) = \frac{x}{1-x}$ is sometimes used to denote the inverse of the function $f(x)$.

The Logarithm as an Inverse

The logarithm is the inverse of the function: $f(x) = 10^x$. You can work this out using the method for finding formulas for inverses that is shown above.

Swap the x and y: $x = 10^y$.

Rearrange to make y the subject: $\log(x) = y \cdot \log(10)$

$\log(x) = y$ (As $\log(10) = 1$—try it on your calculator)

So, if $f(x) = 10^x$, then $f^{-1}(x) = \log(x)$.

When you combine a function and its inverse through composition, you are supposed to get a composite function that gives just x as the output (as the function and inverse "undo" or "reverse" each other's actions). For the function $f(x) = 10^x$ and its inverse $f^{-1}(x) = \log(x)$, this means:

- $10^{\log(x)} = x$.
- $\log(10^x) = x$.

these suggest another meaning for just exactly what the logarithm of a number is:

The logarithm of a number (say 500) is the power to which 10 must be raised in order to produce 500 as the result.

Other useful properties of the logarithm

The logarithm function is very, very helpful when it comes to solving exponential equations like the one you solved for the age of the Desert mummies:

$$0.0000695 = (0.0001) \cdot (0.9998790392)^x$$

to find the age of the mummies, t. When working with exponential equations like the one shown above, it is usually easier to use logarithms to solve for t, rather than to use the GRAPH and INTERSECT capabilities of your calculator.

The important properties of the logarithm function are (A and B are assumed to be positive numbers):

a. $\log(A^T) = T \cdot \log(A)$

b. $\log(A \cdot B) = \log(A) + \log(B)$

c. $\log\left(\frac{A}{B}\right) = \log(A) - \log(B)$

d. $\log(10^T) = T$

e. $10^{\log(A)} = A$