SOLUTIONS

Handout 11: Exponential Functions

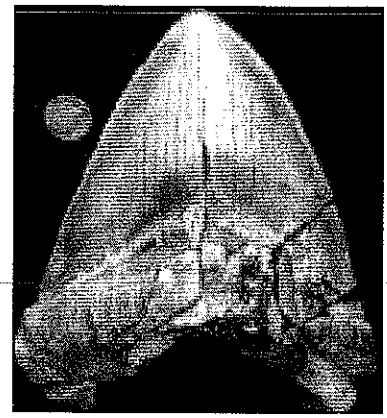

Figure 1: Fossilized *C. megalodon* tooth. (The small circle is a dime.)

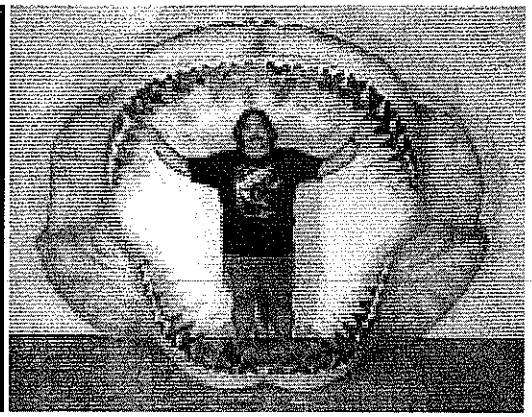

Figure 2: Replica of a *C. megalodon* jaw. (The replica is about seven and a half feet tall.)

The "mega-tooth" shark (*Carcharodon megalodon*) is a giant shark that appears to have lived between 10 and 50 million years ago[1]. Much of what we know about this shark comes from fossilized teeth (see Figures 1 and 2[2]) that have been found in coastal regions of Virginia, North Carolina, South Carolina, Georgia and Florida. Based on the size of these teeth, many scientists believe that *C. megalodon* was approximately the size of a Greyhound bus[3] (see Figures 4(a)[4] and 4(b)[5]).

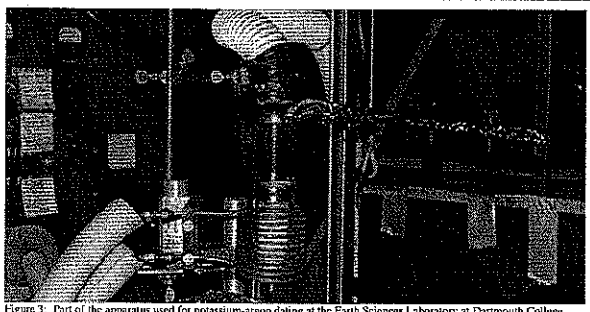

Figure 3: Part of the apparatus used for potassium-argon dating at the Earth Sciences Laboratory at Dartmouth College. This particular item is an induction furnace which is used to extract potassium-40 from the sediment sample.

Many scientists believe that *C. megalodon* died out a long time ago. This view is supported by radioactive dating of the sediments that *C. megalodon* teeth are found in. The specific technique employed for megalodon teeth is called potassium-argon dating.

In this technique, a sample of rock is chemically analyzed and the percentage of the rock that is potassium-40 determined (see Figure 3[6]). Typically for fresh sediment, 2.4% of the sediment is potassium-40. Potassium-40 is a radioactive isotope (half life = 1260 million years) of potassium that decays into the inert gas argon-40. This decay process occurs at a constant percentage rate of change.

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[1] Ellis, R. and McCosker, J. Eds. 1995. *Great White Shark*. Palo Alto, CA: Stanford University Press.
[2] Image source for Figures 1 and 2: http://sharksteeth.com
[3] Gottfried, M.D., L.J.V. Campagno & S.C. Bowman. 1996. Size and skeletal anatomy of the giant "Megatooth" shark *Carcharodon megalodon*. In A.P. Klimley and D.G. Ainley. Eds. *Great White Sharks: The Biology of* Carcharodon carcharias. San Diego, CA: Academic Press.
[4] Image source: http://www.csuohio.edu/internat/new_stu_transport.html
[5] Image source: http://hometown.aol.com/rjravalli/index5.html
[6] Image source: http://www.darmouth.edu/~earthsci/labs/KArlab.htm

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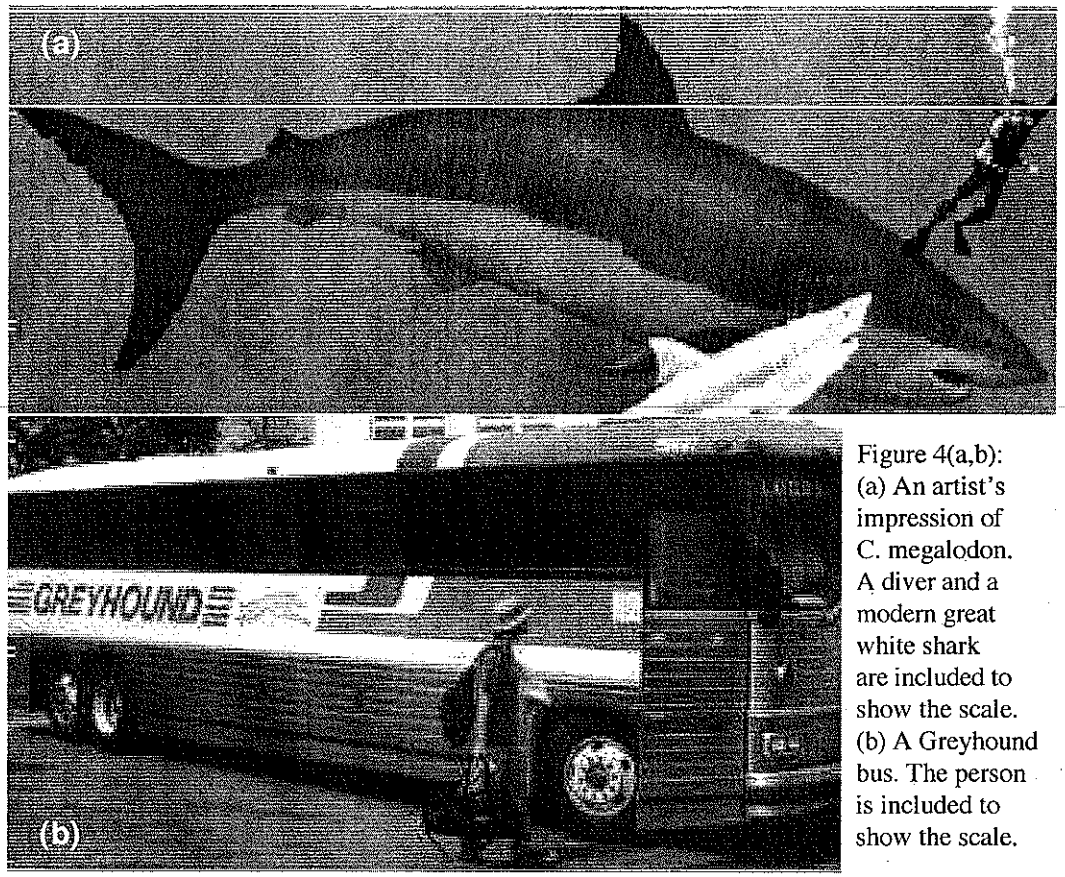

Figure 4(a,b): (a) An artist's impression of C. megalodon. A diver and a modern great white shark are included to show the scale. (b) A Greyhound bus. The person is included to show the scale.

(a) Use the information given on potassium-40 and its decay to complete Table 1 (below).

Age of sediment (millions of years)	0	1260	2520
Percentage of Potassium-40	2.4	1.2	0.6

Table 1.

(b) Is the data recorded in Table 2 perfectly exponential or not?

Yes. The growth factor (if time is measured in millions of years) always comes out to:

$(\frac{1}{2})^{1/1260} = 0.9994500345$

(c) What sort of function will do a good job of representing this situation? Be careful to explain your reasoning.

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(d) Find a formula for the function that gives the percentage of potassium-40 as a function of the age of the rock sample (expressed in millions of years).

Let P = percentage of rock that is potassium-40.
T = age of rock in millions of years.

$$P = (2.4)(0.9994500345)^T$$

(e) A *C. megalodon* tooth was found in a quarry in North Carolina. Lab tests determined that 2.38683% of the sediment that the tooth was found was potassium-40. Set up an equation that would give you the age of the tooth if you were to solve the equation.

$$(2.4)(0.9994500345)^T = 2.38683$$

(f) Use your graphing calculator to find the approximate solution of the equation that you set up in Part (e). Use the viewing window:

Xmin = 0 Xmax = 20 Ymin = 2.3 Ymax = 2.5

$T = 10.0026$ million years.