SOLUTIONS

Handout 10: The Long Shadow of Jack the Ripper

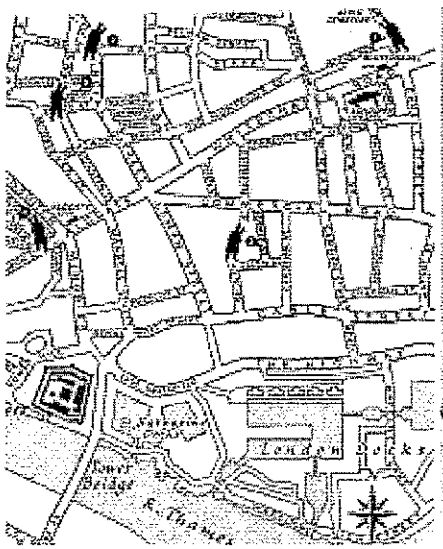

Figure 1: Locations at which the five victims generally attributed to Jack the Ripper were found.

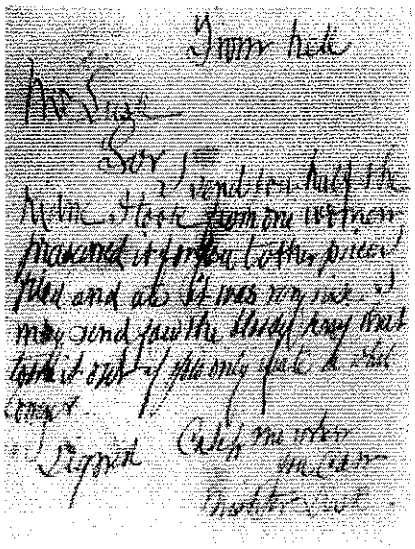

Figure 2: Note sent to police at the time of the Ripper murders. Experts believe that the note was probably *not* written by the killer.

Perhaps the most famous series of unsolved murders in history are the crimes of Jack the Ripper. "Jack the Ripper" is the nickname that has been given to a serial killer who murdered several prostitutes in the East End of London in 1888 (see Figure 1[1]). The nickname of "Jack" originated from a note (see Figure 2) that was written by someone claiming to be the killer, although the note has never been definitely authenticated. At the time of the slayings, the killer was also known as "The Whitechapel Murderer" and "Leather Apron."

Figure 3: A menacing figure appears out of the mist, a long shadow cast by the street lamps.

There is considerable debate over the exact number of prostitutes that Jack the Ripper actually killed. Many authorities believe that one killer was responsible for at least five slayings, although some attribute as many as fourteen deaths to the ripper. The five victims of these brutal murders were:

1. Mary Ann (Polly) Nichols, murdered Friday August 31, 1888.
2. Annie Chapman, murdered Saturday September 8, 1888.
3. Elizabeth Stride, murdered Sunday September 30, 1888.
4. Catherine Eddowes, also murdered Sunday September 20, 1888.
5. Mary Jane Kelly, murdered Friday November 9, 1888. (Played by Heather Graham in the recent movie, "From Hell.")

Perhaps the signature scene of the many "Jack the Ripper" movies and television programs[2] consists of a young woman walking the dark and misty streets of London, her way illuminated by a single street lamp. Suddenly, a male figure in a cape and top hat coalesces from out of the fog. His shadow stretches out, long and ominous before him (see Figure 3[3]). Perhaps sensing danger, the young woman turns. The shadow

[1] Image source: http://www.travelbritain.com/

[2] Legendary British rock group "Spinal Tap" even proposed a rock opera based on the ripper murders. Although the project was never completed, the working title was "Saucy Jack."

[3] Image source: http://www.timbosliverpool.co.uk/

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looms, almost gripping her with sinister, unholy power. Her pretty face contorts in a paroxysm of fear, her lips tremble, too afraid to scream. He is upon her. Darkness descends, and the caped figure dissolves into the mist, phantasmal, leaving death and depravity in its evil wake.

One of the effects that contributes to the atmosphere, suspense and a palpable sense of menace is the way that the murderer's shadow seems to move forward at a much faster rate than the person is actually walking. In this handout, you will use derivatives and the Principle of Similar Triangles to determine why this happens.

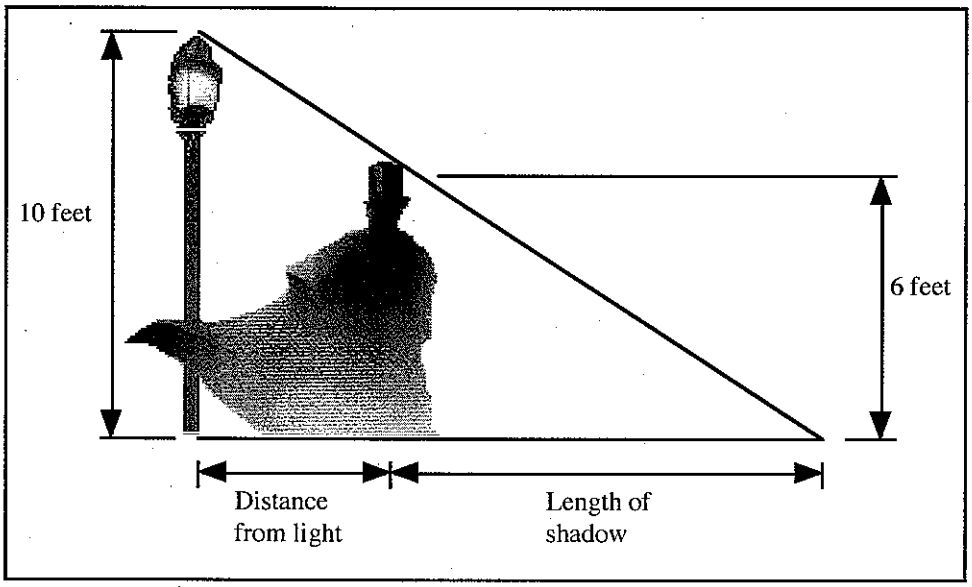
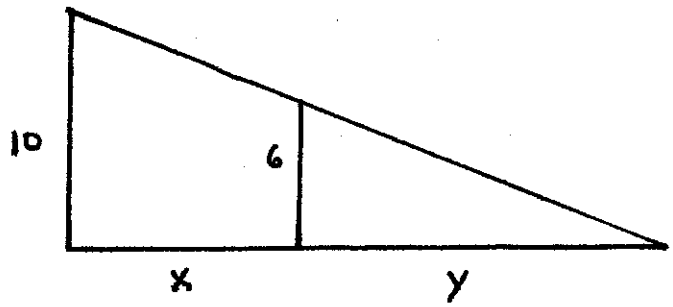


Figure 4: The killer walks away from the street lamp.

1. Figure 4 shows the situation as the killer walks away from the street lamp. The lamp is 18 feet tall, and the killer (including top hat) is 6 feet tall. If x represents the killer's distance from the street lamp in feet and y represents the length of the killer's shadow in feet, find a relationship between x and y.



Using Similar Triangles:

$$\frac{y}{6} = \frac{x+y}{10}$$

So: $$\frac{y}{6} = \frac{x}{10} + \frac{y}{10}$$

$$\frac{y}{15} = \frac{x}{10} \quad \text{or:} \quad y = \frac{3}{2}x$$

2. A normal human walking speed is about 4 feet per second. If this is how fast the killer is walking, how would you represent this fact using the symbols defined in Question 1 and derivatives?

$$\frac{dx}{dt} = 4 \text{ ft/second}$$

3. The ultimate objective of this problem is to find out how quickly the length of the killer's shadow grows. What quantity or derivative do you need to calculate?

$$\frac{dy}{dt}$$

4. Find a relationship between the derivatives $\frac{dx}{dt}$ and $\frac{dy}{dt}$.

$$\frac{dy}{dt} = \frac{3}{2}\frac{dx}{dt}$$

5. How quickly (in units of feet per second) is the killer's shadow growing when he is ten feet from the street lamp?

$$\frac{dy}{dt} = 6 \text{ ft/sec.}$$