MATH 120 – THIRD UNIT TEST

Friday, April 24, 2009.

NAME: SOLUTIONS

Circle the recitation section you attend	Tuesday, Thursday MORNING	Tuesday, Thursday AFTERNOON
	A	B

Instructions:

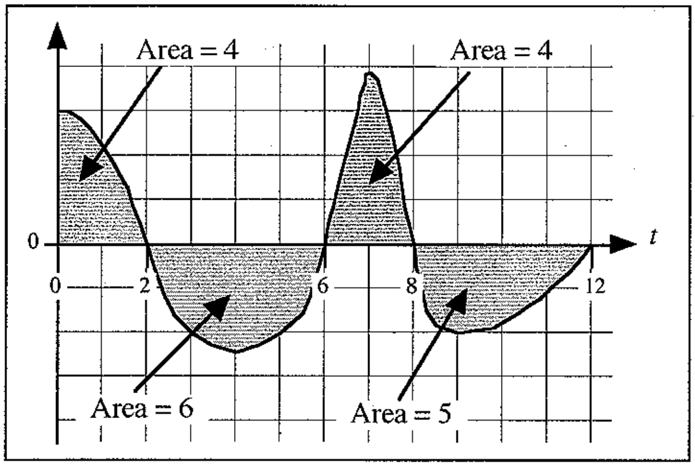
1. Do not separate the pages of the exam. If any pages do become separated, write your name on them and point them out to your instructor when you turn in the exam.
2. Please read the instructions for each individual question carefully. One of the skills being tested on this exam is your ability to interpret questions, so instructors will not answer questions about exam problems during the exam.
3. Show an appropriate amount of work for each exam question so that graders can see your final answer **and** how you obtained it.
4. You may use your calculator on all exam questions except where otherwise indicated. However, if you are asked to find an ***exact*** value of a quantity that involves an integral then you should not use calculator integration for this.
5. If you use graphs or tables to obtain an answer (especially if you create the graphs or tables on your calculator), be certain to provide an explanation and a sketch of the graph to show how you obtained your answer.
6. Please **TURN OFF** all cell phones and pagers, and **REMOVE** all headphones.

Problem	Total	Score
1	14	
2	14	
3	24	
4	13	
5	15	
6	20	
Total	100	

SOLUTIONS

1. 14 Points. CLEARLY INDICATE YOUR FINAL ANSWERS.

In this problem, the function $F(x)$ is defined by the formula $F(x) = \int_0^x f(t)dt$, where $f(t)$ is the function graphed below.

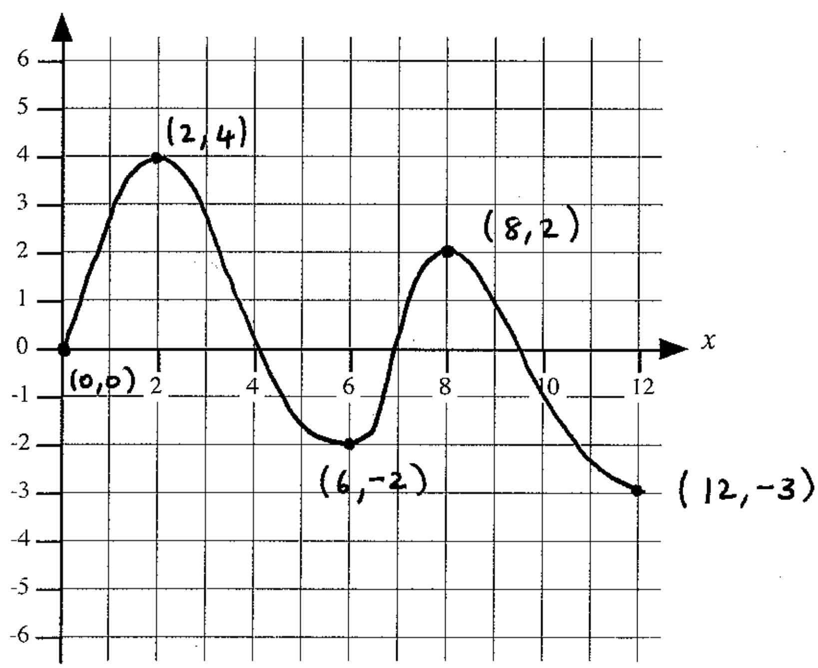


(a) **(8 points)** Find the following function values as accurately as possible:

$F(2) =$ 4 $\qquad$ $F(6) =$ −2

$F(8) =$ 2 $\qquad$ $F(12) =$ −3

(b) **(6 points)** Use the axes provided below to sketch an accurate graph of $y = F(x)$. Label any points on the graph for which you are able to find the exact x- and y-coordinates.



SOLUTIONS

2. 14 Points. SHOW ALL WORK. CLEARLY INDICATE YOUR ANSWERS.

In this problem, the function $f(x)$ will always refer to the function defined by the equation:

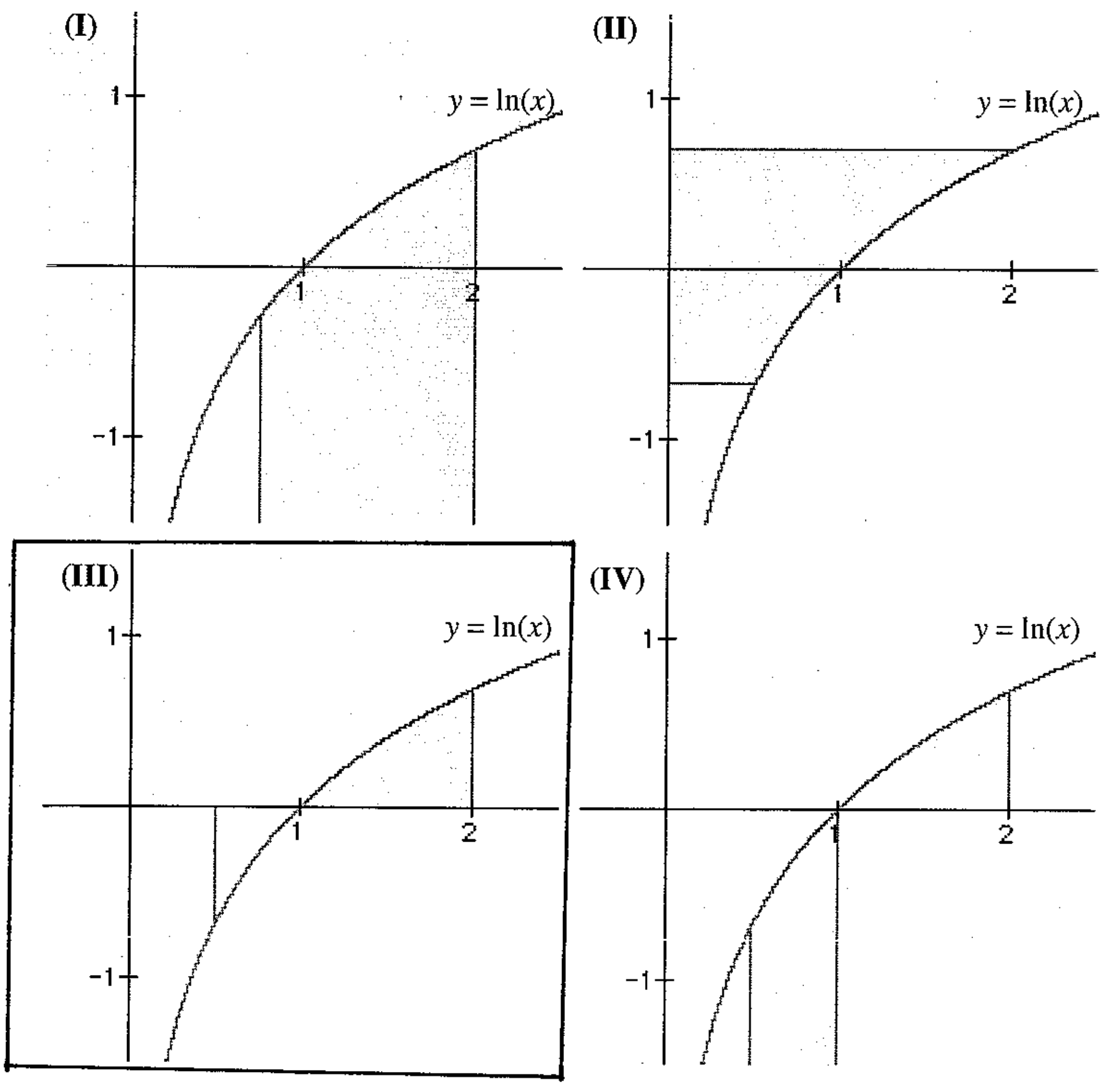
$$f(x) = \ln(x),$$

and the function $F(x)$ will always refer to the function defined by the equation:

$$F(x) = x \cdot \ln(x) - x + 10.$$

(a) **(4 points)** Circle the shaded area (shown below) that most closely corresponds to the symbols:

$$\int_{\frac{1}{2}}^{2} f(x) \cdot dx.$$



Continued on the next page

SOLUTIONS

(b) **(5 points)** Show that the function $F(x)$ is an anti-derivative of $f(x)$.

$$F'(x) = \ln(x) + x \cdot \frac{1}{x} - 1$$

$$= \ln(x) + 1 - 1$$

$$= \ln(x).$$

Since $F'(x) = f(x)$, $F(x)$ is an anti-derivative of $f(x)$.

(c) **(5 points)** Find the numerical value of the integral:

$$\int_{\frac{1}{2}}^{2} f(x) \cdot dx$$

$$\int_{\frac{1}{2}}^{2} f(x)\,dx = F(2) - F(\tfrac{1}{2})$$

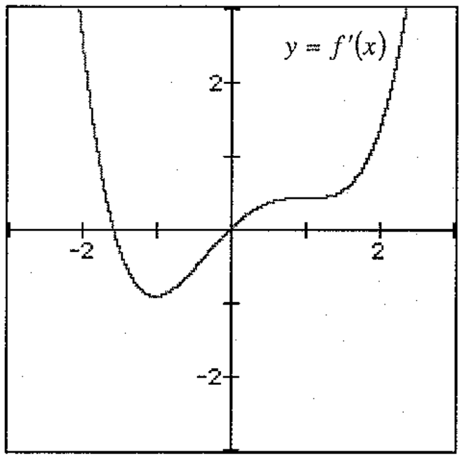
$$= 2 \cdot \ln(2) - 2 + 10 - \left(\tfrac{1}{2} \cdot \ln(\tfrac{1}{2}) - \tfrac{1}{2} + 10\right)$$

$$= 0.2328679514$$

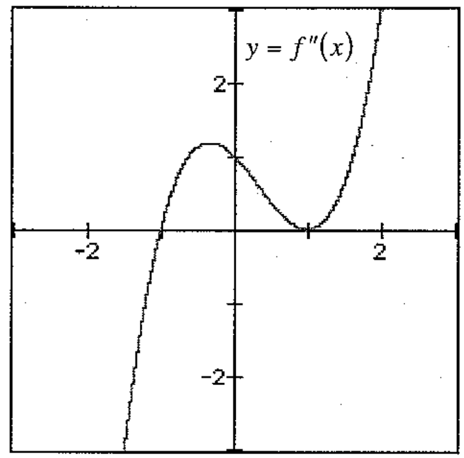
SOLUTIONS

3. 24 Points. CLEARLY INDICATE YOUR ANSWERS.

The graphs given below show the graph of the **derivative**, $y = f'(x)$, and the graph of the **second derivative**, $y = f''(x)$, of a function $f(x)$.



GRAPH OF FIRST DERIVATIVE



GRAPH OF SECOND DERIVATIVE

(a) **(4 points)** Find the x-coordinates of all of the critical points of the function $f(x)$.

The critical points are located at $x = 0$ and $x \approx -1.5$

(b) **(4 points)** Classify each of the critical points as a maximum, minimum or neither.

x	$f''(x)$	Classification.
0	+	Local minimum
−1.5	−	Local maximum.

(c) **(4 points)** Find the x-coordinates of any points at which the concavity of the original function $f(x)$ changes.

The concavity of $f(x)$ changes at $x = -1$.

Continued on the next page.

(d) **(4 points)** Find the x-coordinates of any points at which the second derivative is equal to zero, but the concavity of the original function does not change. Briefly explain how you can tell that the concavity of the original function does not change at the point(s) you have found.

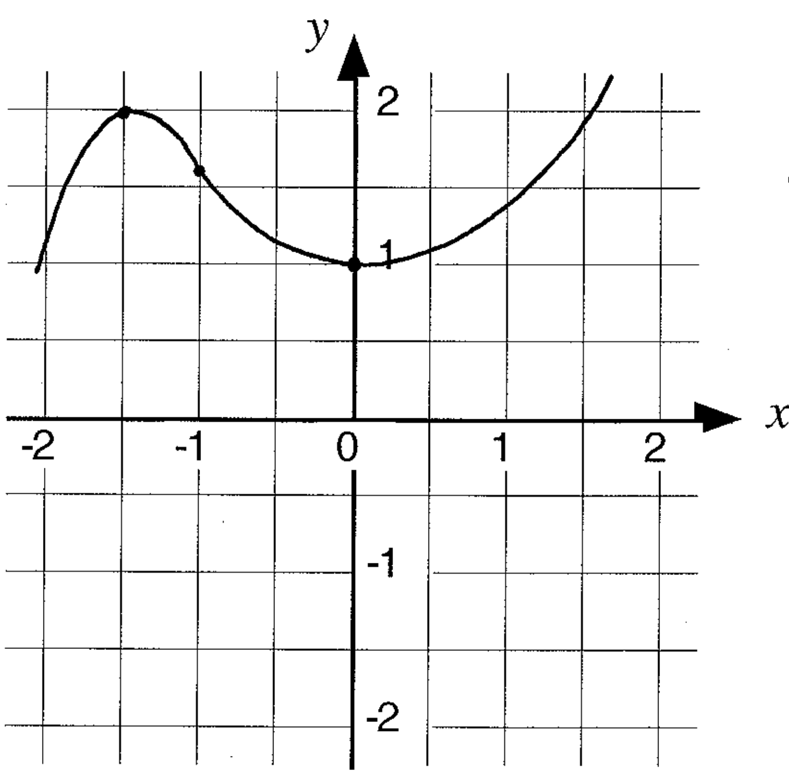
The point in question is $x = 1$.

The concavity does not change because the second derivative does not change sign (i.e. + to − or − to +).

(e) **(8 points)** Suppose that the one other piece of information that you have about the function $f(x)$ is that:

$$f(0) = 1.$$

Use the axes provided below to sketch a graph of $y = f(x)$ that is consistent with the information that you have determined in Parts (a)-(d).



(Many answers are possible here.)

SOLUTIONS

4. **13 Points. CLEARLY INDICATE YOUR FINAL ANSWERS.**

(a) **(5 points)** Find the approximate value of $\int_1^4 e^{\sqrt{x}}\,dx$ obtained when the integral is approximated using a left-hand Riemann sum and 50 rectangles.

$$\Delta x = \frac{4-1}{50} = 0.06$$

$$\text{Left hand Riemann sum} = \sum_{K=0}^{49} e^{\sqrt{1+K(0.06)}}(0.06) = 14.63813541$$

(b) **(5 points)** Find the approximate value of $\int_1^4 e^{\sqrt{x}}\,dx$ obtained when the integral is approximated using a right-hand Riemann sum and 50 rectangles.

$$\text{Right hand Riemann sum} = \sum_{k=1}^{50} e^{\sqrt{1+K(0.06)}}(0.06) = 14.91838187.$$

(c) **(3 points)** Find the best estimate of $\int_1^4 e^{\sqrt{x}}\,dx$ obtained when the integral is approximated using a 50 rectangles.

$$\text{Best estimate} = \frac{14.63813541 + 14.91838187}{2}$$

$$= 14.77825864.$$

5. 15 Points. NO WORK = NO CREDIT. CLEARLY INDICATE FINAL ANSWER.

Use **Newton's method** to find the solution of the equation:

$$\sin(x) = 1 - x.$$

correct to eight (8) decimal places. In order to get credit, you must show your work.

You should not use your calculator on this problem for anything except simple arithmetic, and evaluating trigonometric functions. In particular, you should not solve the equation by tracing the graph of a function or funding intersection points on your calculator.

If you use your calculator, make sure it is set in RADIAN mode. Clearly indicate your final answer and include at least eight (8) decimal places.

Here $f(x) = \sin(x) - 1 + x$ and

$f'(x) = \cos(x) + 1.$

We will start with the guess of $x_1 = 0.5$.

n	x_n	$f(x_n)$	$f'(x_n)$	x_{n+1}
1	0.5	-0.0205744614	1.877582562	0.5109577953
2	0.5109577953	$-2.89760902 \times 10^{-5}$	1.872276456	0.5109734294
3	0.5109734294	-5.856×10^{-11}	1.872268888	0.5109734294
4				
5				

As the two values obtained for x_3 and x_4 agree to eight decimal places, the solution of the equation $\sin(x) = 1 - x$ will be:

$$x = 0.5109734294.$$

SOLUTIONS

6. 20 Points. NO WORK = NO CREDIT. CLEARLY INDICATE YOUR ANSWERS.

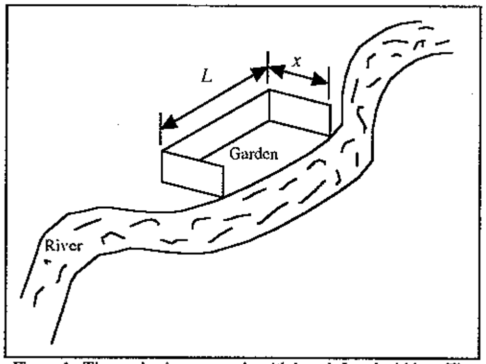


Figure 1: The garden is a rectangle with length L and width x. The river provides one of the sides of the garden. Fences are built on the other three sides of the garden.

A garden is built beside a river. The shape of the garden is a **rectangle** with length L and width x. Fences are built to form three sides of the garden. The river forms the last side of the garden. **Both of the short pieces of fence are x feet long.**

The person building the garden has 100 feet of fence to form the three sides of the garden.

The person building the garden wants to make a garden that **has the largest possible area.**

Your ultimate goal in this problem is to work out the value of x that will make the area of the garden as large as possible.

(a) **(5 points)** The person building the garden has 100 feet of fence to form the three sides of the garden. Write down an equation involving x, L and numbers to express this situation using mathematical symbols.

$$2x + L = 100$$

$$L = 100 - 2x$$

(b) **(5 points)** The area of the garden is equal to length times width: Area = $x \cdot L$. Use the equation that you set up in Part (a) to eliminate the letter L from this area formula. Your final answer should consist of an area formula that only involves the letter x and numbers.

$$\begin{aligned} \text{Area} &= x \cdot L \\ &= x(100 - 2x) \\ &= 100x - 2x^2 \end{aligned}$$

Continued on the next page.

NO WORK = NO CREDIT. CLEARLY INDICATE YOUR ANSWERS.

(c) **(6 points)** Find the value of x that will make the area of the garden as large as possible. Show your work. As part of your answer you should demonstrate that your value of x actually maximizes the area.

$$A(x) = 100x - 2x^2$$

$$A'(x) = 100 - 4x = 0$$

Solution is : $x = 25$ feet.

To show that $x = 25$ maximizes area, use the Second Derivative test.

$$A''(x) = -4 < 0,$$

(d) **(4 points)** What is the largest possible area that the garden can have? Include appropriate units with your answer.

$$A(25) = 1250 \text{ ft}^2.$$