

MATH 120 – SECOND UNIT TEST

Friday, March 20, 2009.

NAME: _____

Circle the recitation
section you attend

Tuesday, Thursday
MORNING

Tuesday, Thursday
AFTERNOON

A

B

Instructions:

1. Do not separate the pages of the exam. If any pages do become separated, write your name on them and point them out to your instructor when you turn in the exam.
2. Please read the instructions for each individual question carefully. One of the skills being tested on this exam is your ability to interpret questions, so instructors will not answer questions about exam problems during the exam.
3. Show an appropriate amount of work for each exam question so that graders can see your final answer **and** how you obtained it.
4. You may use your calculator on all exam questions except where otherwise indicated. However, if you are asked to find an *exact* value of a quantity that involves an integral then you should not use calculator integration for this.
5. If you use graphs or tables to obtain an answer (especially if you create the graphs or tables on your calculator), be certain to provide an explanation and a sketch of the graph to show how you obtained your answer.
6. Please **TURN OFF** all cell phones and pagers, and **REMOVE** all headphones.

Problem	Total	Score
1	13	
2	19	
3	14	
4	15	
5	15	
6	24	
Total	100	

1. 13 Points. SHOW WORK AND CLEARLY INDICATE FINAL ANSWERS.

The Folium of Descartes is a curve with the equation:

$$x^3 + y^3 = 3 \cdot x \cdot y.$$

(a) **(8 points)** Find a formula for $\frac{dy}{dx}$ for the Folium of Descartes.

(b) **(5 points)** Find the equation of the tangent line to the Folium of Descartes at the point $\left(\frac{3}{2}, \frac{3}{2}\right)$.

2. 19 Points. SHOW ALL WORK. CLEARLY INDICATE YOUR ANSWERS.

In this problem, the function $f(x)$ will always refer to:

$$f(x) = \frac{x+1}{2x+1}.$$

- (a) **(7 points)** Find a formula for $f^{-1}(y)$. As part of your answer, you should demonstrate that such a formula exists.

- (b) **(3 points)** Evaluate $f^{-1}\left(\frac{2}{3}\right)$.

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SHOW ALL WORK. CLEARLY INDICATE YOUR ANSWERS.

In this problem, the function $f(x)$ will always refer to:

$$f(x) = \frac{x+1}{2x+1}.$$

(c) **(4 points)** Evaluate $f'(1)$.

(d) **(5 points)** Write down an equation for the tangent line to the curve $s = f^{-1}(t)$ at the point where $t = \frac{2}{3}$.

3. 14 Points. SHOW YOUR WORK.

Carbon-14 is a naturally occurring radioactive element with a half life of 5730 years. This means that if you have 100 micrograms (μg) of carbon-14 today, after 5730 years, you will have only 50 μg of carbon-14 left.

- (a) **(3 points)** Let $M(t)$ represent the mass (in μg) of carbon-14 present in a piece of wood. $M(t)$ can be defined by a differential equation:

$$M'(t) = k \cdot M(t).$$

Write down a formula for the function $M(t)$. Your formula may contain up to two unspecified constants.

- (b) **(4 points)** The half life of carbon-14 is 5730 years. Use this to evaluate one of the constants in the formula that you found in Part (a). Include at least eight (8) decimal places with your answer.

- (c) **(7 points)** A piece of living wood normally contains 0.001 μg of carbon-14. When the wood dies, the carbon-14 begins to decay. A piece of wood found buried at Stonehenge in England contained only 0.00063 μg of carbon-14. How old is this piece of wood? Include at least two (2) decimal places with your answer.

4. 15 Points. SHOW ALL WORK. CLEARLY INDICATE YOUR FINAL ANSWER.

A lighthouse is located on a small island, four (4) miles from the nearest point P on a straight shoreline. The light in the lighthouse makes two (2) revolutions per minute. How fast is the beam of light moving along the shoreline when it is half a mile from P ?

Show your work (no work = no credit) and clearly write your final answer in the space provided below. Include appropriate units with your answer. If you give your final answer as a decimal include at least four (4) decimal places.

SPEED OF BEAM OF LIGHT ALONG SHORE: _____

5. 15 Points. NO WORK = NO CREDIT. CLEARLY INDICATE FINAL ANSWERS.

Solve each of the following equations to find all possible solutions, x . You may use the following formula without justifying it:

$$\log_a(x) = \frac{\ln(x)}{\ln(a)} \quad a > 0.$$

You should not use your calculator on this problem for anything except simple arithmetic, evaluating logarithms and exponentials. In particular, you should not solve these equations by tracing the graph of a function or finding intersection points on your calculator.

(a) (9 points) $\log(x + 1) - \log(x) = 2.$

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NO WORK = NO CREDIT. CLEARLY INDICATE FINAL ANSWERS.

Solve each of the following equations to find all possible solutions, x . You may use the following formula without justifying it:

$$\log_a(x) = \frac{\ln(x)}{\ln(a)} \quad a > 0.$$

You should not use your calculator on this problem for anything except simple arithmetic, evaluating logarithms and exponentials. In particular, you should not solve these equations by tracing the graph of a function or finding intersection points on your calculator.

(b) (6 points) $\log(x) = \ln(x)$.

6. 24 Points. NO WORK = NO CREDIT. CLEARLY INDICATE FINAL ANSWERS.

In each case, determine whether the limit exists. If the limit exists, find its value. You may use the following results without having to justify them:

$$\sinh(0) = 0$$

$$\cosh(0) = 1.$$

(a) (8 points) $\lim_{x \rightarrow 0} \frac{1 - \cosh(5x)}{x^2}.$

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NO WORK = NO CREDIT. CLEARLY INDICATE FINAL ANSWERS.

In each case, determine whether the limit exists. If the limit exists, find its value.

(b) (8 points)
$$\lim_{x \rightarrow \infty} \frac{\ln(\ln(x))}{x \cdot \ln(x)} .$$

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NO WORK = NO CREDIT. CLEARLY INDICATE FINAL ANSWERS.

In each case, determine whether the limit exists. If the limit exists, find its value.

(c) **(8 points)** $\lim_{x \rightarrow 1} \left(\frac{1}{\ln(x)} - \frac{1}{x-1} \right)$.