MATH 120 – FIRST UNIT TEST

Friday, February 13, 2009.

NAME: SOLUTIONS

Circle the recitation section you attend	Tuesday, Thursday MORNING	Tuesday, Thursday AFTERNOON
	A	B

Instructions:

1. Do not separate the pages of the exam. If any pages do become separated, write your name on them and point them out to your instructor when you turn in the exam.
2. Please read the instructions for each individual question carefully. One of the skills being tested on this exam is your ability to interpret questions, so instructors will not answer questions about exam problems during the exam.
3. Show an appropriate amount of work for each exam question so that graders can see your final answer **and** how you obtained it.
4. You may use your calculator on all exam questions except where otherwise indicated. However, if you are asked to find an ***exact*** value of a quantity that involves an integral then you should not use calculator integration for this.
5. If you use graphs or tables to obtain an answer (especially if you create the graphs or tables on your calculator), be certain to provide an explanation and a sketch of the graph to show how you obtained your answer.
6. Please **TURN OFF** all cell phones and pagers, and **REMOVE** all headphones.

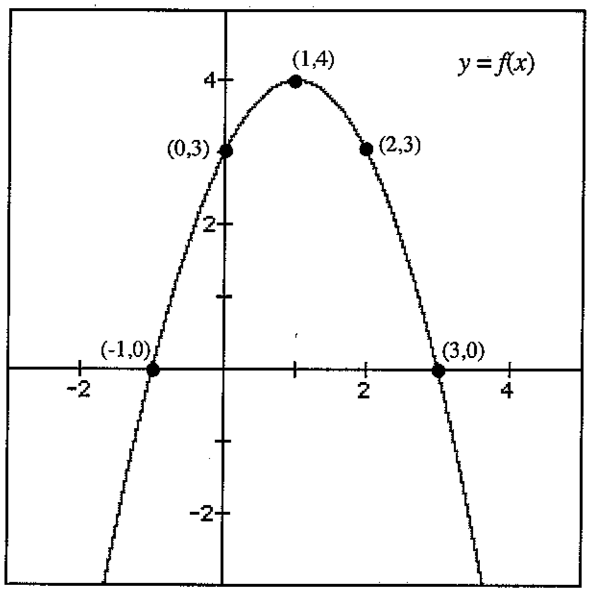
Problem	Total	Score
1	19	
2	21	
3	20	
4	11	
5	14	
6	15	
Total	100	

SOLUTIONS

1. 19 Points. CLEARLY INDICATE ANSWERS. NO PARTIAL CREDIT FOR (a)-(c).

The quadratic function $f(x)$ is defined by the graph shown below. The function $g(x)$ is defined by the formula:

$$g(x) = x^2 - 1.$$



(a) **(3 points)** Evaluate: $\frac{f(2)}{g(2)}$. $= \frac{3}{3} = 1.$

(b) **(3 points)** Evaluate: $\frac{f(3)-f(1)}{3-1}$. $= \frac{0 - 4}{3 - 1} = -2$

(c) **(3 points)** Evaluate: $f(g(1))$. $= f(0) = 3.$

(d) **(5 points)** Find the DOMAIN of the function: $m(x) = \sqrt{f(x)}$.

Domain: $-1 \leq x \leq 3$

(e) **(5 points)** Find the DOMAIN of the function: $n(x) = \frac{f(x)}{g(x)}$.

Domain: All real numbers except $x = -1$ and $x = +1$.

SOLUTIONS

2. **21 Points. SHOW ALL WORK. NO PARTIAL CREDIT WITHOUT WORK.**

In this problem, all that you may assume that and $g(x)$ are differentiable functions, the domains of both functions include $-4 \le x \le 4$ and that they have the values shown below.

$f(2)=1$ $\quad f(4)=3 \quad g(3)=4 \quad g(4)=2.$

$f'(2)=5$ $\quad f'(4)=7 \quad g'(3)=8 \quad g'(4)=6.$

(a) **(5 points)** Find the exact value of the derivative $k'(4)$ where $k(x)=f(x)\cdot g(x)$.

Product Rule: $k'(x) = f'(x)\cdot g(x) + f(x)\cdot g'(x)$

$$k'(4) = f'(4)\cdot g(4) + f(4)\cdot g'(4)$$
$$= (7)(2) + (3)(6)$$
$$= 14 + 18$$
$$= 32.$$

(b) **(6 points)** Find the exact value of the derivative $p'(4)$ where $p(x)=\dfrac{f(x)}{g(x)}$.

Quotient Rule: $p'(x) = \dfrac{f'(x)\cdot g(x) - g'(x)\cdot f(x)}{g(x)^2}$

$$p'(4) = \frac{f'(4)\cdot g(4) - g'(4)\cdot f(4)}{g(4)^2}$$
$$= \frac{(7)(2) - (6)(3)}{(2)^2}$$
$$= \frac{14 - 18}{4}$$

Continued on the next page.

$$= -1.$$

In this problem, all that you may assume that and $g(x)$ are differentiable functions, the domains of both functions include $-4 \le x \le 4$ and that they have the values shown below.

$$f(2)=1 \qquad f(4)=3 \qquad g(3)=4 \qquad g(4)=2.$$

$$f'(2)=5 \qquad f'(4)=7 \qquad g'(3)=8 \qquad g'(4)=6.$$

(c) **(5 points)** Find the exact value of the derivative $q'(4)$ where $q(x)=f(g(x))$.

Chain Rule: $q'(x) = f'(g(x)) \cdot g'(x)$

$$q'(4) = f'(2) \cdot g'(4)$$

$$= (5)(6)$$

$$= 30.$$

(d) **(5 points)** Find the exact value of the derivative $m'(4)$ where $m(x)=g(f(x))$.

Chain Rule: $m'(x) = g'(f(x)) \cdot f'(x)$

$$m'(4) = g'(3) \cdot f'(4)$$

$$= (8)(7)$$

$$= 56.$$

SOLUTIONS

3. 20 Points. SHOW YOUR WORK. NO WORK = NO CREDIT.

In each part of this problem, use algebra to calculate the limit. Clearly indicate the value of the limit that represents your final answer. You should not just plug numbers into your calculator to guess the value of a limit.

If you believe that a limit does not exist, write "DOES NOT EXIST" and show why the limit does not exist (for example, using a graph).

(a) **(10 points)** $\lim_{x \to -4} \dfrac{\frac{1}{4}+\frac{1}{x}}{4+x}$.

$$\lim_{x \to -4} \frac{1/4 + 1/x}{4+x} = \lim_{x \to -4} \frac{\frac{x+4}{4x}}{4+x}$$

$$= \lim_{x \to -4} \frac{1}{4x}$$

$$= \frac{-1}{16}$$

Continued on the next page.

In each part of this problem, use algebra to calculate the limit. Clearly indicate the value of the limit that represents your final answer. You should not just plug numbers into your calculator to guess the value of a limit.

If you believe that a limit does not exist, write "DOES NOT EXIST" and show why the limit does not exist (for example, using a graph).

(b) **(10 points)** $\lim\limits_{x \to \infty} \dfrac{1-2x^2-x^4}{5+x-3x^4}$.

Dominator of Denominator is x^4

$$\lim_{x \to \infty} \frac{1-2x^2-x^4}{5+x-3x^4} = \lim_{x \to \infty} \frac{\frac{1}{x^4} - \frac{2x^2}{x^4} - \frac{x^4}{x^4}}{\frac{5}{x^4} + \frac{x}{x^4} - \frac{3x^4}{x^4}}$$

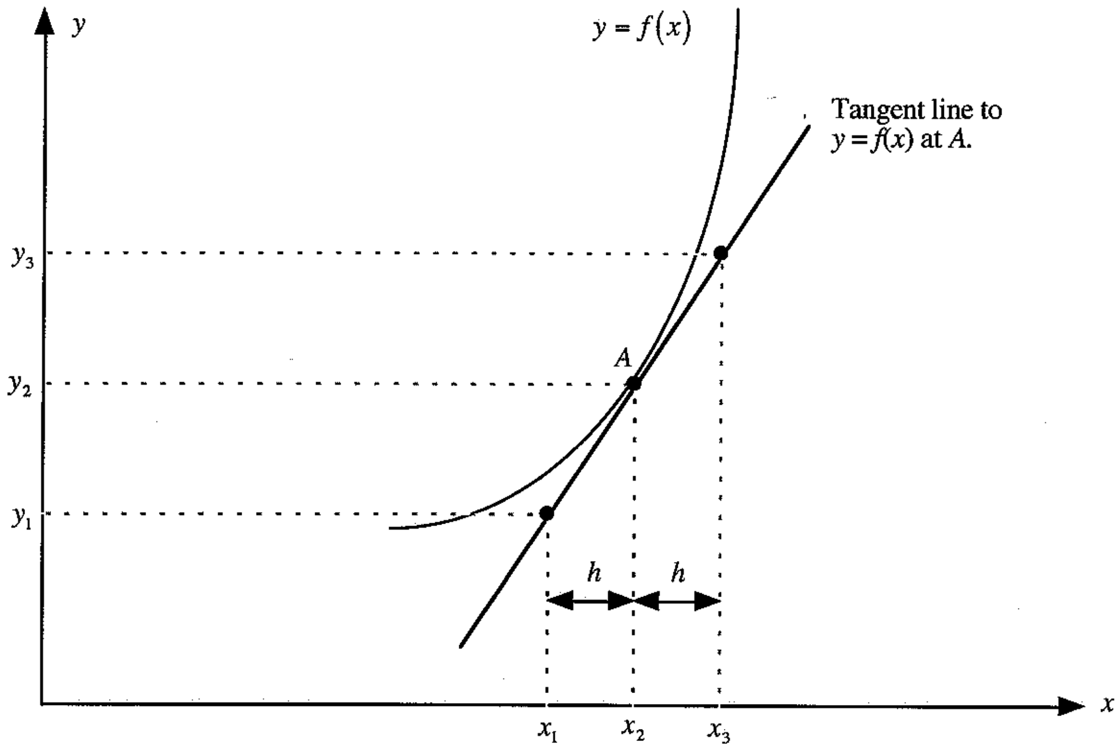
$$= \lim_{x \to \infty} \frac{\frac{1}{x^4} - \frac{2}{x^2} - 1}{\frac{5}{x^4} + \frac{1}{x^3} - 3}$$

$$= \frac{1}{3}$$

SOLUTIONS

4. 11 Points.

The graph given below shows the graph of $y = f(x)$. (The graph is not drawn perfectly to scale, so do not rely on measurements from the graph to answer this question.) The point A on this graph corresponds to $x = 1$.



In addition to what you can see from the graph, you may assume that:

$$f(1) = 3 \qquad f'(1) = 2 \qquad h = 0.1.$$

(a) **(5 points)** Find an equation for the tangent line that touches the graph $y = f(x)$ at the point A.

$$y - 3 = 2(x - 1)$$

or $$y = 2x + 1$$

(b) **(6 points)** Find the exact numerical values of each of the quantities indicated below. Write your answers in the spaces provided below.

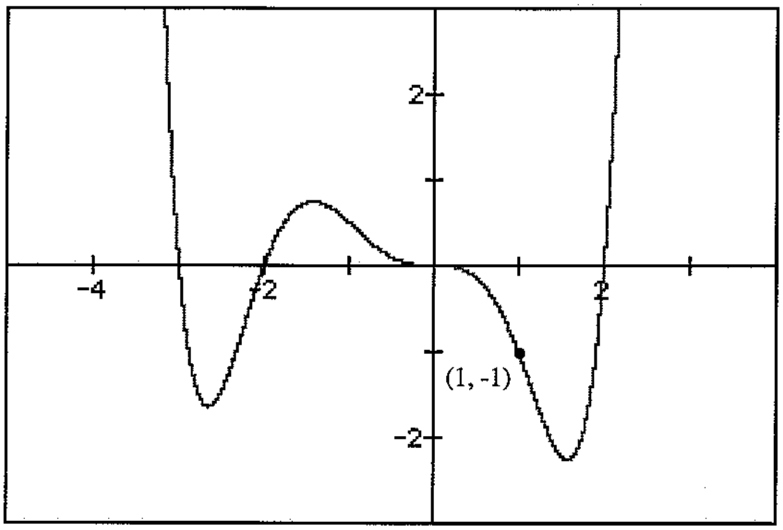
$x_1 =$ 0.9 $\qquad x_2 =$ 1

$x_3 =$ 1.1 $\qquad y_1 =$ 2.8

$y_2 =$ 3 $\qquad y_3 =$ 3.2

5. **14 Points. SHOW ALL WORK. NO WORK = NO CREDIT.**

Your ultimate goal in this question is to find an equation for the polynomial function $f(x)$ whose graph is shown below. The only letters that should appear in your final answer are $f(x)$ and x.



(a) **(8 points)** Find the ALL of the roots and multiplicities of the function $f(x)$. Record your answers in the table shown below.

Root	Multiplicity
-3	1
-2	1
0	3
2	1

(b) **(6 points)** Use your answers to Part (a) of this problem to find a formula for the polynomial function $f(x)$.

$$y = f(x) = k \cdot (x+3) \cdot (x+2) \cdot (x-0)^3 \cdot (x-2)$$

To find k, plug in $y=-1$ and $x=1$.

$$-1 = k \cdot (4)(3)(1)^3(-1) \quad \text{so} \quad k = \frac{1}{12}$$

Final Answer: $f(x) = \frac{1}{12}(x+3)(x+2)(x-0)^3(x-2)$

6. **15 Points. SHOW ALL WORK. NO WORK = NO CREDIT.**

In each case, use the **definition of the derivative** to calculate a formula for $f'(x)$.

You should not use any "short cut" rules that you may know to calculate the formula for $f'(x)$ here. Clearly indicate your final answer.

(a) **(6 points)** $f(x) = x^2$

Difference Quotient :

$$\frac{f(x+h) - f(x)}{h} = \frac{(x+h)^2 - x^2}{h}$$

$$= \frac{x^2 + 2xh + h^2 - x^2}{h}$$

$$= \frac{h \cdot (2x + h)}{h}$$

$$= 2x + h \quad , \quad h \neq 0.$$

Taking the limit as $h \to 0$ gives $f'(x)$:

$$f'(x) = \lim_{h \to 0} 2x + h = 2x.$$

Continued on the next page.

In each case, use the **definition of the derivative** to calculate a formula for $f'(x)$.

You should not use any "short cut" rules that you may know to calculate the formula for $f'(x)$ here. Clearly indicate your final answer.

(b) **(9 points)** $f(x) = \sqrt{1+2x}$

Difference Quotient:

$$\frac{f(x+h) - f(x)}{h} = \frac{\sqrt{1+2(x+h)} - \sqrt{1+2x}}{h}$$

Now, $\left(\sqrt{1+2(x+h)} - \sqrt{1+2x}\right)\left(\sqrt{1+2(x+h)} + \sqrt{1+2x}\right)$

$$= 1 + 2(x+h) - (1+2x)$$

$$= 1 + 2x + 2h - 1 - 2x$$

$$= 2h.$$

So:

$$\frac{f(x+h) - f(x)}{h} = \frac{2h}{h\left(\sqrt{1+2(x+h)} + \sqrt{1+2x}\right)}$$

$$= \frac{2}{\sqrt{1+2(x+h)} + \sqrt{1+2x}} \qquad h \neq 0$$

Taking the limit of the difference quotient as $h \to 0$ gives $f'(x)$:

$$f'(x) = \lim_{h \to 0} \frac{2}{\sqrt{1+2(x+h)} + \sqrt{1+2x}} = \frac{1}{\sqrt{1+2x}}.$$