MATH 120 – FINAL EXAM

Friday, May 8, 2009.

NAME: ____

Circle the recitation section you attend	Tuesday, Thursday MORNING	Tuesday, Thursday AFTERNOON
	A	B

Instructions:

1. Do not separate the pages of the exam.
2. Please read the instructions for each individual question carefully.
3. Show an appropriate amount of work for each exam question so that graders can see your final answer **and** how you obtained it.
4. You may use your calculator on all exam questions except where otherwise indicated.
5. If you use graphs or tables to obtain an answer (especially if you create the graphs or tables on your calculator), be certain to provide an explanation and a sketch of the graph to show how you obtained your answer.
6. Be sure to use appropriate algebraic and limit notation.
7. **TURN OFF** all cell phones and pagers, and **REMOVE** all headphones.

Problem	Total	Score
1	6	
2	8	
3	10	
4	8	
5	6	
6	12	
7	9	
8	7	
9	12	
10	12	
11	10	
Total	**100**	

1. 6 points. SHOW YOUR WORK.

Carbon-14 is a radioactive substance with a half-life of 5730 years. A living human thigh bone contains 3 micrograms of carbon-14. When a person dies, the carbon-14 in their thigh bone begins to decay exponentially.

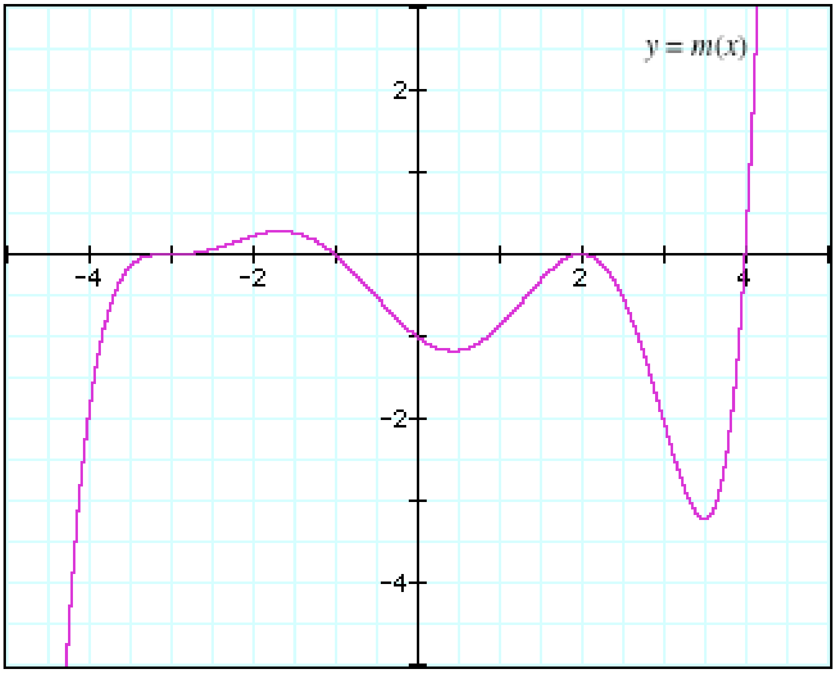
(a) **(2 points)** Write down a formula that gives C, the number of micrograms of carbon-14 in a human thigh bone from a person who has been dead for T years.

(b) **(4 points)** A human thigh bone from an archaeological site in Western China was found to have 2.08 micrograms of carbon-14 in it. How many years ago did the person die? If you give your answer as a decimal, include at least four (4) decimal places.

NOTE: In order to get credit for this problem, you must solve the problem using algebra and SHOW YOUR WORK. If you solve the problem on your calculator or fail to show your work, then you will get zero credit, even if your answer is correct.

2. 8 Points. CLEARLY INDICATE YOUR FINAL ANSWER.

Your ultimate goal in this question is to find an equation for the polynomial function $m(x)$ whose graph is shown below. The equation that you give as a final answer should contain no unspecified constants (such as k).



(a) **(2 points)** Find the x-coordinates of ALL zeros of the function $m(x)$.

(b) **(2 points)** Find the MULTIPLICITY of each of the zeros of the function $m(x)$.

(c) **(4 points)** Use your answers to Parts (a) and (b) of this problem to find a formula for the polynomial function $m(x)$. Write your final answer in the space provided below.

ANSWER: $m(x)$ = ____

3. 10 Points. CLEARLY INDICATE YOUR ANSWERS.

In this problem, the functions $f(x)$ and $g(x)$ will always refer to the functions defined by the formulas:

$$f(x) = \sqrt{x+2} \qquad \text{and} \qquad g(x) = -(x-1)\cdot(x-2).$$

The domain of $f(x)$ consists of all numbers greater than or equal to −2 and the domain of $g(x)$ consists of all numbers.

(a) **(2 points)** Write down a formula for the COMPOSITE FUNCTION $(f \circ g)(x)$.

$(f \circ g)(x)$ = ______________________________

(b) **(2 points)** Write down a formula for the COMPOSITE FUNCTION $(g \circ f)(x)$.

$(g \circ f)(x)$ = ______________________________

(c) **(4 points)** What is the DOMAIN of the composite function: $(f \circ g)(x)$? You can express your answer in any mathematically valid format. Two possibilities are as an interval or as an inequality.

(d) **(2 points)** What is the DOMAIN of the composite function: $(g \circ f)(x)$? You can express your answer in any mathematically valid format. Two possibilities are as an interval or as an inequality.

4. 8 Points. SHOW YOUR WORK. CLEARLY INDICATE FINAL ANSWERS.

Let $C(t)$ be the concentration of a drug in a patient's bloodstream t hours after the drug is administered. The units of $C(t)$ are mg/dl (milligrams per deciliter). As the patient's body eliminates the drug, $C(t)$ decreases at a rate that is proportional to the amount of the drug that remains in the patient's bloodstream so that:

$$\frac{dC}{dt} = C'(t) = -k \cdot C(t),$$

where k is a positive constant called the elimination constant.

(a) **(2 points)** Suppose that the initial (at $t = 0$) concentration of the drug is 20 mg/dl. Find a formula for $C(t)$. Your formula should contain at most one unspecified constant.

(b) **(3 points)** The patient's body eliminates half of the drug in 30 hours. Find the value of the elimination constant, k. Include at least four (4) decimal places in your answer.

(c) **(3 points)** How many hours does it take for the patient's body to eliminate 90% of the drug? Include at least four (4) decimal places in your answer.

5. 6 points total. SHOW YOUR WORK.

Use the **definition of the derivative** to calculate a formula for $f'(x)$. You should not use any "short cut" rules that you may know to calculate the formula for $f'(x)$ here. Clearly indicate your final answer.

$$f(x) = 5x^2 + \frac{2}{\sqrt{x}}$$

6. 12 points total. SHOW YOUR WORK. NO WORK = NO CREDIT.

In each case, determine whether the limit exists. If the limit exists, find its value.

(a) (3 points) $\lim_{t \to 0} \left(\frac{1}{t} - \frac{1}{e^t - 1} \right).$

(b) (3 points) $\lim_{t \to 0} \frac{\sin^2(A \cdot t)}{\cos(A \cdot t) - 1}$, where $A \neq 0$.

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In each case, determine whether the limit exists. If the limit exists, find its value.

(c) **(3 points)** $\lim_{x \to 0^+} x^a \cdot \ln(x)$, where $a > 0$.

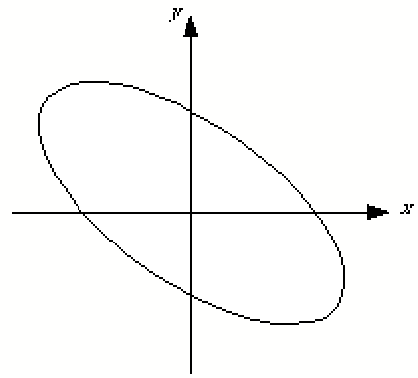
(d) **(3 points)** $\lim_{x \to 0} \dfrac{\cos(x)}{x}$.

7. 9 Points. SHOW YOUR WORK.

The equation

$$x^2 + 2xy + 3y^2 = 2$$

defines the elliptical curve shown below.



(a) **(3 points)** Find a formula for $\frac{dy}{dx}$.

(b) **(2 points)** Find a formula for the tangent line to the ellipse that touches the ellipse at the point:

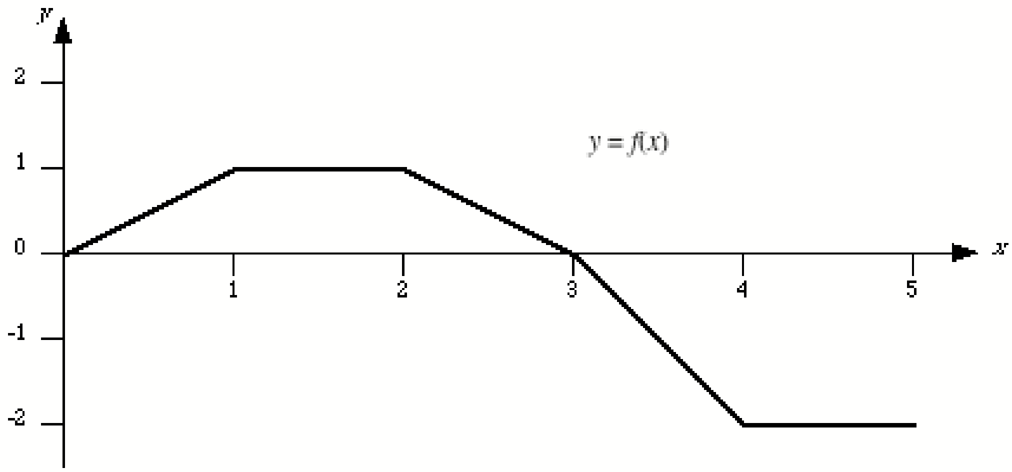
$$(x, y) = \left(0, \sqrt{\tfrac{2}{3}}\right).$$

(c) **(4 points)** Find the coordinates (x and y) of all points on the ellipse where the tangent line is **horizontal**. Record your answers in the table provided below. If you give any answers as decimals, include at least four (4) decimal places.

x	y

8. 7 Points. CLEARLY INDICATE YOUR ANSWERS.

In this problem you will be studying a function called $F(x)$. The derivative of $F(x)$ is called $f(x)$, i.e. $F'(x) = f(x)$. The graph below is a graph of $y = f(x)$.



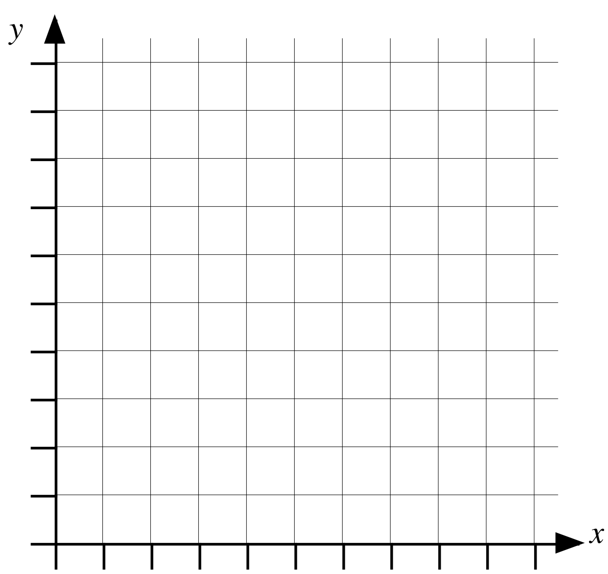
Suppose that $F(5) = 7$. Use the information provided to answer the following questions.

(a) **(4 points)** What are the values of $F(1)$, $F(2)$, $F(3)$, and $F(4)$?

$F(1) =$ ______________________ $F(2) =$ ______________________

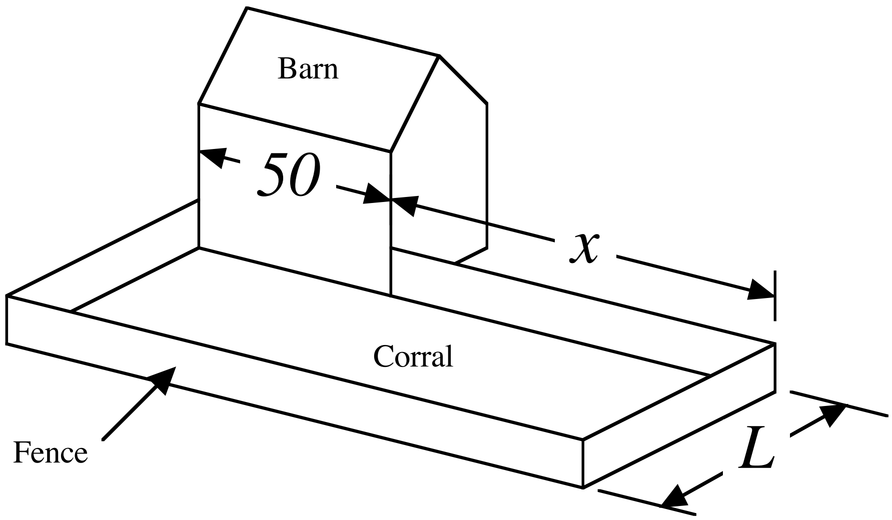
$F(3) =$ ______________________ $F(4) =$ ______________________

(b) **(3 points)** Use the axes provided below to sketch an accurate graph of $y = F(x)$. Be careful to label any points on the graph whose coordinates you know exactly.



9. 12 Points. SHOW ALL WORK. NO PARTIAL CREDIT WITHOUT WORK.

A horse breeder is planning to build a corral next to her barn. Her barn is 50 feet long and the side of the barn will form part of one side of the corral. The horse breeder has 250 feet of fencing material to work with and wishes to make the area of the corral as large as possible.



(a) **(2 points)** The person building the corral has 250 feet of fence to form the sides of the corral. Write down an equation involving x, L and numbers to express this situation using mathematical symbols.

(b) **(3 points)** The area of the corral is equal to length times width. Find a formula for the area of the corral that involves only the variable x and constants.

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NO WORK = NO CREDIT. CLEARLY INDICATE YOUR ANSWERS.

(c) **(5 points)** Find the value of x that will make the area of the corral as large as possible. Show your work. As part of your answer you should demonstrate that your value of x actually maximizes the area.

(d) **(2 points)** What is the largest possible area that the corral can have? Include appropriate units with your answer.

10. 12 Points. SHOW ALL WORK. NO CREDIT WITHOUT WORK.

In this problem you are required to calculate formulas for each of the anti-derivatives (or indefinite integrals) listed below. Your answers should each include an unspecified constant along the lines of "+*C*."

NOTE: You should not use your calculator to evaluate integrals in this problem, apart from working out arithmetic and evaluating functions.

(a) (3 points) $$\int \frac{\sec^2(x)}{\tan(x)} \cdot dx$$

(b) (3 points) $$\int \frac{2x}{1+x^4} \cdot dx$$

Continued on the next page.

In this problem you are required to calculate formulas for each of the anti-derivatives (or indefinite integrals) listed below. Your answers should each include an unspecified constant along the lines of "+*C*."

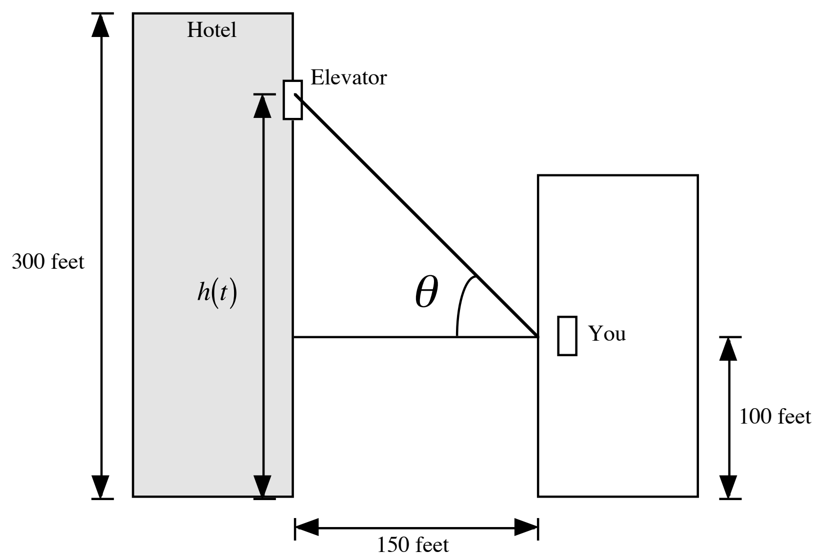
NOTE: You should not use your calculator to evaluate integrals in this problem, apart from working out arithmetic and evaluating functions.

(c) **(3 points)** $\int x \cdot e^{x} \cdot dx$

(d) **(3 points)** $\int x \cdot \ln(x) \cdot dx$

11. 10 Points. SHOW YOUR WORK. NO WORK = NO CREDIT.

For the entertainment of its guests, a hotel has a glass elevator that goes up and down the outside of the building (see diagram given below). The hotel is 300 feet tall.



You are watching the elevator from a room in a building located 150 feet from the hotel. Your room is 100 feet from the ground. At time $t = 0$, the elevator is at the top of the hotel and immediately begins to descend at a constant speed of 30 feet per second. Let θ represent the angle between your horizon and your line of sight to the elevator.

(a) **(2 points)** Find a formula for $h(t)$, the elevator's height above the ground as it descends from the top of the hotel.

(b) **(3 points)** Use your answer from Part (a) to express θ as a function of time t and find the rate of change of θ with respect to time.

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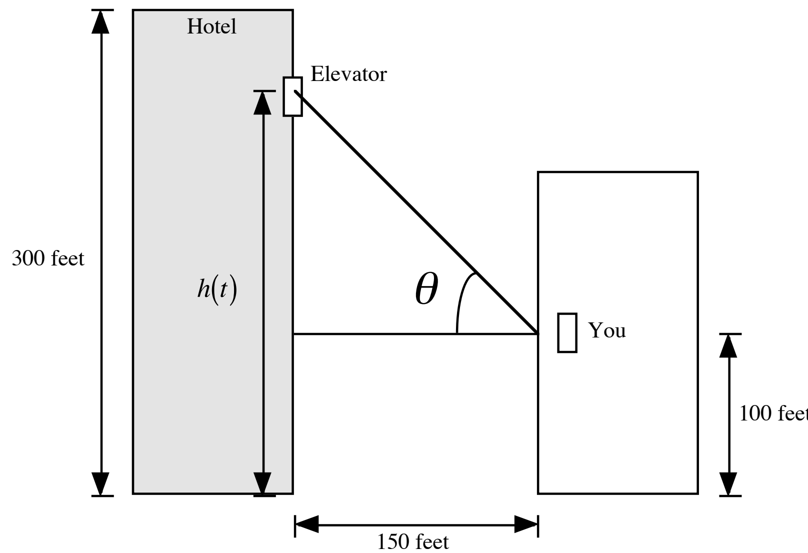
For the entertainment of its guests, a hotel has a glass elevator that goes up and down the outside of the building (see diagram given below). The hotel is 300 feet tall.



You are watching the elevator from a room in a building located 150 feet from the hotel. Your room is 100 feet from the ground. At time $t = 0$, the elevator is at the top of the hotel and immediately begins to descend at a constant speed of 30 feet per second. Let θ represent the angle between your horizon and your line of sight to the elevator.

(c) **(5 points)** The rate of change of θ with respect to time is a measure of how fast the elevator appears to be moving as you watch it. At what height above the ground is the elevator when it appears to be moving the most quickly to you?