Department of Mathematics
CARNEGIE MELLON UNIVERSITY
Professor Doytchinov
21-256 Multivariate Analysis and Approximation, Fall 1998

Review for Test 4

For Test 4 you must be familiar with the following topics and be able to solve problems related to them.

- geometric sequences and series;
- geometric growth: population dynamics, interest bearing accounts, etc.;
- compound interest and savings;
- present value;
- mortgages and amortization
- power series; Taylor and McLaurin series.

The test will have 6 problems. Five of the problems will be worth 18 points each and will be pretty similar to the homework problems and the in-class examples. The sixth problem will be worth 10 points and will be a "surprize" problem, i.e., it will require some creativity for its solution.

When preparing for the test, first review *all homework problems and in-class examples.* Below are some more sample problems similar to the ones to appear on the test. The problems marked with a * give you an idea of what the "surprize" problems are like.

You will need a calculator for Test 4. Any scientific calculator with the ability to compute logarithms and powers will suffice. Calculators with QWERTY keybord are not allowed.

Review Problems

1. * Compute the sum
$$1+\frac{2}{3}+\frac{3}{3^2}+\frac{4}{3^3}+\cdots+\frac{n}{3^{n-1}}.$$

2. * Let $0<r<1$. Find the value of
$$S=1+2r+3r^2+\cdots+nr^{n-1}+\cdots$$
as a function of r.

3. * Consider the following periodic decimal fractions:
$$\begin{aligned} a &= 8.04848484848\ldots \\ b &= 0.00404040404\ldots \end{aligned}$$
Compute ab and a/b.

4. * Let $a=\underbrace{1111\cdots111}_{1998 \text{ ones}}$, and $b=\underbrace{2222\cdots222}_{999 \text{ twos}}$. Compute $\sqrt{a-b}$.

5. The population of a city is 20000, and increases by 7% each year. After how many years will the population double?

6. In a far-off country, there exist 400 animals of a certain unique endangered species. It is estimated, that each year the the existing animals give offspring that is numerically equal to 25% of the previous year's population, while 5% of the previous year's population dies from natural causes. It is also estimated, that each year 100 animals are killed by illegal local hunters for their valuable fur. After how many years will the species become extinct?

7. A certain savings account pays 8%, compounded annually. At the beginning, \$5000 are deposited. Further, deposits are made at the end of each year, increasing the amount of the deposit by 2%. Give a general formula for the amount accumulated at the end of the n^{th} year.

8. In order to generate income for 15 years, \$300000 are deposited into a savings account paying 7% compounded annually. If a constant amount of money is to be withdrawn at the end of each of the 15 years, how large can this amount be?

9. Determine the monthly payment on a 30-year mortgage with a loan amount of \$240000 and annual percentage rate 12%, compounded monthly.

10. Use Taylor's formula to find the Taylor series for $f(x)$ at the given value of a.

    (a) $f(x) = \cos x, \quad a = \pi/4,$

    (b) $f(x) = \sin x, \quad a = -\pi/4,$

    (c) $f(x) = \sqrt{x}, \quad a = 9,$

    (d) $f(x) = \ln x, \quad a = 2.$

11. Obtain the Maclaurin series for the given function by manipulating standard Maclaurin series decompositions given on the formula sheet.

    (a) $f(x) = e^{2x},$

    (b) $f(x) = e^{x+2},$

    (c) $f(x) = x^2 \cos(x^3),$

    (d) $f(x) = \ln \sqrt{\frac{1+x}{1-x}}.$

12. Evaluate the indefinite integral as a power series:

    (a) $\int e^{x^3}\, dx,$

    (b) $\int \frac{\sin x}{x}\, dx,$

    (c) $\int \frac{\arctan x}{x}\, dx,$

    (d) $\int \frac{x}{1-x^7}\, dx.$

13. Compute the following sums as functions of x:

    (a) $1 - x^2 + x^4 - x^6 + \cdots + (-1)^n x^{2n} + \cdots,$

    (b) $1 + 2x + 3x^2 + 4x^3 + \cdots + (n+1)x^n + \cdots,$

    (c) $1 - x^4 + \frac{x^8}{2!} - \frac{x^{12}}{3!} + \cdots + (-1)^n \frac{x^{4n}}{n!} + \cdots.$