Department of Mathematics
CARNEGIE MELLON UNIVERSITY
Professor Doytchinov
21-256 Multivariate Analysis and Approximation, Spring 1998

Review for Test 2

For Test 2 you must be familiar with the following topics and be able to solve problems related to them.

- Functions of several variables, graphs, level curves and level surfaces;
- limits of functions of several variables;
- continuity of functions of several variables;
- partial derivatives;
- Clairaut's Theorem;
- equation of a tangent plane to the surface $z = f(x, y)$;
- differentiability of functions of several variables;
- differential;
- the chain rule;
- implicit differentiation.

The test will have 6 problems. Five of the problems will be worth 18 points each and will be pretty similar to the homework problems and the in-class examples. The sixth problem will be worth 10 points and will be a "surprize" problem, i.e., it will require some creativity for its solution.

Below are some sample problems similar to the ones to appear on the test. The problems marked with a * give you an idea of what the "surprize" problems are like. Also, be sure to review all homework problems.

Review Problems

1. Find the domain and the range of the function:

   (**a**) $f(x, y) = \sqrt{\ln(x - y)}$ (**b**) $f(x, y, z) = \ln(64 - 16x^2 - 9y^2 - 25z^2)$.

2. Determine the largest set on which the function is continuous:

   (**a**) $f(x, y) = \begin{cases} \frac{xy^2}{x^2+2y^2}, & \text{if } (x, y) \neq (0, 0), \\ 0, & \text{if } (x, y) = (0, 0). \end{cases}$ (**b**) $f(x, y) = \begin{cases} \frac{x^2-y^2}{x^2+y^2}, & \text{if } (x, y) \neq (0, 0), \\ 0, & \text{if } (x, y) = (0, 0). \end{cases}$

3. Find the limit, or show that the limit does not exist.

   (**a**) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^3}{x^2 + y^2}$ (**b**) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^3}{x\,y^2 + y^3}$

4. Let $f(x,y) = \sin(x+3y) + x^2y^3 + 2y^2$. Compute f_x, f_y, f_{xx}, f_{xy}, and f_{yy}.

5. * For what values of r is the function

$$f(x,y,z) = \begin{cases} \frac{(x+y+z)^r}{x^2+y^2+z^2}, & \text{if } (x,y,z) \neq (0,0,0), \\ 0, & \text{if } (x,y,z) = (0,0,0). \end{cases}$$

continuous on $\mathbb{R}^3$?

6. Find the tangent plane and normal line to the surface $z = x^2 + 4y^2$ at the point $(-1, 2, 17)$.

7. Suppose that $w = x^3y + y^2 - 4z^3, x = rs, y = (r^2 - s^2)^3$, and $z = 2r + s$. Use the chain rule to compute $\frac{\partial w}{\partial s}$. Express your answer in terms o r and s alone. Algebraic simplification is not necessary.

8. * For

$$f(x,y,z) = \frac{x}{x^2+y^2} e^{\sin(x^2y)} + \sin(z-2)\ln(x + \sqrt{1+x^2}),$$

compute $f_x(1,0,2)$.

9. Let f be a continuously differentiable function of one variable and define

$$w(x,y) := f\left(\frac{y}{y-x}\right).$$

Show that, for $x \neq y$:

$$x\,\frac{\partial w}{\partial x} + y\,\frac{\partial w}{\partial y} = 0.$$

10. * For

$$f(x,y) = 5 + 6(x-3) + 8(y+1) + \sin(x-3)^2 + (y+1)\ln\sqrt{2+y},$$

write an equaion of the tangent plane to the graph at the point $(3, -1, 5)$.

11. Assume that u and v have continuous second partial derivatives and that

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \qquad \text{and} \qquad \frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y}.$$

Show that

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \qquad \text{and} \qquad \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = 0.$$

12. The equation $xyz = \cos(x+y+z)$ defines implicitly z as a function of x and y. Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.