Formula Sheet:

- **Tangent Planes:**
  - An equation of the tangent plane to the surface $z = f(x, y)$ at the point (x_0, y_0, z_0) is
    $$z - z_0 = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0).$$
  - An equation of the tangent plane to the surface $F(x, y, z) = 0$ at the point (x_0, y_0, z_0) is
    $$F_x(x_0, y_0, z_0)(x-x_0)+F_y(x_0, y_0, z_0)(y-y_0)+F_z(x_0, y_0, z_0)(z-z_0) = 0.$$
- **Kuhn-Tucker:** Let $(x_1^*, \ldots, x_n^*)$ be a solution of the following optimization problem:

$$\begin{aligned} f(x_1, \ldots, x_n) &\to \max \\ \text{Subject to:} & \\ g_1(x_1, \ldots, x_n) &= 0, \\ &\vdots \\ g_m(x_1, \ldots, x_n) &= 0, \\ h_1(x_1, \ldots, x_n) &\leq 0, \\ &\vdots \\ h_s(x_1, \ldots, x_n) &\leq 0. \end{aligned}$$

Define the Lagrange function

$$\begin{aligned} & L(x_1, \ldots, x_n, \lambda_1, \ldots, \lambda_m, \mu_1, \ldots, \mu_s) \\ &= f(x_1, \ldots, x_n) - \sum_{i=1}^{m} \lambda_i g_i(x_1, \ldots, x_n) - \sum_{j=1}^{s} \mu_j h_j(x_1, \ldots, x_n). \end{aligned}$$

Then there exist real numbers $\lambda_1, \ldots, \lambda_m, \mu_1, \ldots, \mu_s$ such that

$$\begin{aligned} \frac{\partial}{\partial x_k} L(x_1^*, \ldots, x_n^*, \lambda_1, \ldots, \lambda_m, \mu_1, \ldots, \mu_s) &= 0 \text{ for } k = 1, \ldots, n \\ g_i(x_1^*, \ldots, x_n^*) &= 0 \text{ for } i = 1, \ldots, m \\ \mu_j h_j(x_1^*, \ldots, x_n^*) &= 0 \text{ for } j = 1, \ldots, s \\ h_j(x_1^*, \ldots, x_n^*) &\leq 0 \text{ for } j = 1, \ldots, s \\ \mu_j &\geq 0 \text{ for } i = 1, \ldots, s. \end{aligned}$$