Department of Mathematics
CARNEGIE MELLON UNIVERSITY
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21-259 Calulus in 3D, Spring 1999

Review for Test 3

For Test 3 you must be familiar with the following topics and be able to solve problems related to them:

- Double and iterated integrals;
- Fubini's theorem; change of order of integration;
- areas and double integrals in polar coordinates;
- triple integrals; change of order of integration;
- triple integrals in spherical and cylindrical coordinates;
- change of variables in multiple integrals.

The test will have 7 problems. Six of the problems will be worth 15 points each and will be similar to the homework problems and the in-class examples. The last problem will be worth 10 points and will be a "surprize" problem, i.e., it will require some creativity for its solution.

Below are some sample problems similar to the ones to appear on the test. The problems marked with a * give you an idea what the "surprize" problems are like. Also, be sure to review all homework problems.

Review Problems

1. Write an equivalent double integral with the order of integration reversed.

$$(\mathbf{a})\quad \int_0^1\int_y^{\sqrt{y}} f(x,y)\,dx\,dy,\qquad (\mathbf{b})\quad \int_0^2\int_1^{e^{2x}} f(x,y)\,dy\,dx,\qquad (\mathbf{c})\quad \int_0^2\int_{2x}^4 f(x,y)\,dy\,dx.$$

2. Evaluate each of the following

$$(\mathbf{a})\quad \int_0^2\int_{-\sqrt{4-x^2}}^0 e^{x^2+y^2}\,dy\,dx,\qquad\qquad (\mathbf{b})\quad \int_0^1\int_x^1 3x\sqrt{x^2+y^2}\,dy\,dx.$$

3. Compute

$$\iint_D (x^2+y^2)(1+\sqrt{x^2+y^2})\,dx\,dy,$$

where D is the region in the first quadrant that is boubded by $y=\sqrt{1-x^2}, y=x$, and the x-axis.

4. * Let $f(x,y)$ be defined and continuous for all $(x,y)\in\mathbb{R}^2$. For every $t>0$ consider the region

$$D_t := \{(x,y) : x^2+y^2 \le t^2\}$$

and define

$$F(t) := \iint_{D_t} f(x,y)\,dA.$$

Show that, for $t > 0$, the derivative of F is given by

$$F'(t) = \int_0^{2\pi} t\,f(t\cos\theta, t\sin\theta)\,d\theta.$$

5. Find the volume of the region enclosed by $y = x^2$, $y = 1$, $z = 0$, and $z = 2$.

6. Let E be the region in the first octant that is bounded by the coordinate planes, the plane $x + y = 1$, and the plane $z = 3$. Evaluate

$$\iiint_E (1 + 2z)\,dV.$$

7. Find the volume of the region that is inside the sphere $x^2 + y^2 + z^2 = 4$, but outside the cylinder $x^2 + y^2 = 1$.

8. Given

$$\int_0^1 \int_{-3}^3 \int_{x^2}^9 f(x,y,z)\,dz\,dx\,dy,$$

write the equivalent triple integral in the order $dy\,dx\,dz$.

9. Let E be the region that is bounded below by the cone $z = \sqrt{x^2 + y^2}$ and above by the sphere $x^2 + y^2 + z^2 = 1$. Evaluate

$$\iiint_E z^3 \sqrt{x^2 + y^2 + z^2}\,dV.$$

10. * Let f be continuous on $[0,1]$ and let R be the triangular region with vertices $(0,0), (1,0)$, and $(0,1)$. Show that

$$\iint_R f(x+y)\,dA = \int_0^1 u f(u)\,du.$$

11. * Compute the integral

$$\iint_R \cos\left(\frac{y-x}{y+x}\right)\,dA,$$

where R is the trapezoidal region with vertices $(1,0)$, $(2,0)$, $(0,2)$, and $(0,1)$.