# A Numerical Study of the 'One and One Half' Dimensional Vlasov-Maxwell Sytem

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Jack Schaeffer
Department of Mathematics Sciences
Carnegie Mellon University
Pittsburgh, PA 15213
e-mail: js5m@andrew.cmu.edu

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**Abstract**

We consider the Cauchy problem for the Vlasov-Maxwell system in a low dimensional setting. It is not known if solutions that start smooth remain smooth. To gain some insight into this question, the problem is solved with both a finite difference method and a particle method. An attempt is made to choose conditions which will result in a solution that is likely to blow up. However, when both methods are used, it is found that the spatial derivatives of the fields do not grow rapidly.

## 1 Introduction

Consider the one and one half dimensional Vlasov-Maxwell system:

$$\begin{cases} \partial_t f + v_1 \partial_x f + (E_1 + v_2 B)\partial_{v_1} f + (E_2 - v_1 B)\partial_{v_2} f = 0 \\ \partial_x E_1 = \rho = \int f \, dv - b(x) \\ \partial_t E_1 = -j_1 = -\int f v_1 dv \\ \partial_t E_2 = -\partial_x B - j_2 \\ \partial_t B = -\partial_x E_2 \end{cases} \tag{1.1}$$

Here $t$ is time, $x \in \mathbb{R}$ is position, $v = (v_1, v_2) \in \mathbb{R}^2$ is velocity, and $f(t, x, v)$ is the number density in phase space of particles of charge one and mass one. A fixed background of neutralizing charge is given by $b(x)$ so $\rho(t, x)$ is the charge density and

$$j(t,x) = \int_{\mathbb{R}^s} f(t,x,v) v \, dv$$

is the current density. The induced electromagnetic field is given by

$$\overrightarrow{E} = (E_1, E_2, 0) \; \overrightarrow{B} = (0,0,B).$$

The speed of light has been taken to be one.

Let $f^{(0)} : \mathbb{R}^3 \to [0,\infty)$, $E_2^{(0)} : \mathbb{R} \to \mathbb{R}$, $B^{(0)} : \mathbb{R} \to \mathbb{R}$, and $b : \mathbb{R} \to \mathbb{R}$ be continuously differentiable and compactly supported with

$$\iint f^{(0)} dv \, dx = \int b \, dx \tag{1.2}$$

Define

$$E_1^{(0)}(x) = \int_{-\infty}^{x} \left( \int f^0 dv - b \right) d\tilde{x}.$$

We take

$$(f, E_1, E_2, B)\Big|_{t=0} = (f^{(0)}, E_1^{(0)}.E_2^{(0)}, B^{(0)}) \tag{1.3}$$

as the initial condition for (1.1).

This paper is motivated by the following question: does the solution $(f, E_1, E_2, B)$ of (1.1) with (1.3) remain smooth for all $t \geq 0$? This question has attracted some interest but has remained open. Hence, we will try to gain some insight by use of numerical experimentation. (1.1) can be compared with the relativistic Vlasov-Maxwell system:

$$\begin{cases} \partial_t f + \hat{v}_1 \partial_x f + (E_1 + \hat{v}_2 B)\partial_{v_1} f + (E_2 - \hat{v}_1 B)\partial_{v_2} f = 0 \\ \partial_x E_1 = \rho = \displaystyle\int f \, dv - b \\ \partial_t E_1 = -j_1 = -\displaystyle\int f \hat{v}_1 \, dv \\ \partial_t E_2 = -\partial_x B - j_2 = -\partial_x B - \displaystyle\int f \hat{v}_2 \, dv \\ \partial_t B = -\partial_x E_2. \end{cases} \tag{1.4}$$

Here $v$ is momentum and

$$\hat{v} = \frac{v}{\sqrt{1+|v|^2}}$$

is velocity. A crucial difference between (1.1) and (1.4) is that for (1.4)

$$|\hat{v}| < 1 = \text{ speed of light}$$

for all $v$. In contrast, for (1.1) transport speed equal the speed of light may occur and may even be present in the initial conditions. For (1.4) it is known [8] that solutions remain smooth for all time. A crucial aspect of this result comes from [11] where it is shown for the three dimensional version of (1.4) that solutions remain smooth as long as the $v$ support of $f$ remains bounded. These bounds are not known in three dimensions but have been obtained in two dimensions [9], [10].

In contrast, for (1.1) bounds on $v$ support are known [7], but bounds on derivatives of $f$ have not been obtained. This is because the decomposition of derivatives technique used in [8] and [11] becomes singular at $v_1 = \pm 1$ for (1.1). It is shown in [7] that if a solution of (1.1) breaks down then the first singularity must appear at $v_1$ equal to one or minus one.

We mention that in [2, 5, 14, 15, 16, 20, 22] the issue described above is avoided by considering a model which includes diffusion in $v$. We also mention that global weak solutions of (1.1) in three dimensions are constructed in [4]. For the Vlasov-Poisson system, global existence of smooth solutions is known in three dimensions [12, 13, 17, 19]. For general references on related problems see [6] and [18].

The problem (1.1) possesses an invariant which we will use. Define

$$A = \int_{-\infty}^{x} B(t, \tilde{x}) d\tilde{x}$$

and

$$G(t, x, v_1, w) = f(t, x, v_1, w - A(t, x)).$$

Then

$$\partial_t G + v_1 \partial_x G + (E_1 + (w - A)\partial_x A)\partial_{v_1} G = 0$$

and

$$j_2 = \iint G(t, x, v_1, w)(w - A) dw\, dv_1. \tag{1.5}$$

In order to reduce the computational size of the problem we consider solutions of the form

$$G(t, x, v_1, w) = g(t, x, v_1)\delta(w - w_0)$$

for some $w_0 \in \mathbb{R}$. Now the problem becomes

$$\begin{cases} \partial_t g + v_1 \partial_x g + (E_1 + (w_0 - A)\partial_x A)\partial_{v_1} g = 0 \\ E_1 = \int_{-\infty}^{x} \left( \int g\, dv_1 - b \right) d\tilde{x} \\ \partial_t^2 A - \partial_x^2 A = j_2 = (w_0 - A) \int g\, dv_1. \end{cases} \tag{1.6}$$

We wish to choose initial conditions which lead to a solution which breaks down. Let us examine the conditions that are likely to occur in a solution that breaks down and then build these into the initial conditions. Define

$$K = E_1 + (w_0 - A)B$$

and $(X(s,t,x,v),\ V_1(s,t,x,v))$ by

$$\begin{aligned} \frac{dX}{ds} &= V_1 & X(t,t,x,v_1) &= x \\ \frac{dV_1}{ds} &= K(s,X) & V_1(t,t,x,v_1) &= v_1. \end{aligned}$$

Then

$$\frac{d}{ds}\left[\partial_x g(s,X,V_1)\right] = -\partial_x K(s,X)\partial_{v_1} g(s,X,V_1)$$

and

$$\frac{d}{ds}\left[\partial_{v_1} g(s,X,V_1)\right] = -\partial_x g(s,X,V_1). \tag{1.7}$$

Suppose that for some $x, v_1$, as $s \to T^-$

$$\left|\partial_x g(s,X,V_1)\right| \to +\infty,\ \left|\partial_{v_1} g(s,X,V_1)\right| \to +\infty. \tag{1.8}$$

In fact, $|\partial_x g|$ and $|\partial_{v_1} g|$ must be unbounded for blowup to occur, but this does not immediately imply (1.8). In light of (1.7) we expect that

$$\partial_x g(s,X,V_1)\ \partial_{v_1} g(s,X,V_1) < 0$$

on some interval $s \in [t_1, T)$. Then

$$\begin{aligned} & -\int_{t_1}^{t} \frac{\partial_x g(s,X,V_1)}{\partial_{v_1} g(s,X,V_1)}\, ds \\ = \quad & \ln\left|\partial_{v_1} g(s,X,V_1)\right|\Big|_{t_1}^{t} \to +\infty \end{aligned}$$

as $t \to T^-$ so we expect

$$\frac{-\partial_x g(s,X,V_1)}{\partial_{v_1} g(s,X,V_1)} \to +\infty$$

as $s \to T^-$. Furthermore, as $t \to T^-$

$$\begin{aligned} & \int_{t_1}^{t} \left[\partial_x K(s,X) - \left(\frac{\partial_x g(s,X,V)}{\partial_{v_1} g(s,X,V)}\right)^2\right] ds \\ = \quad & \frac{-\partial_x g(s,X,V)}{\partial_{v_1} g(s,X,V)}\Bigg|_{t_1}^{t} \to +\infty \end{aligned}$$

so we also expect

$$\partial_x K(s,X) - \left(\frac{\partial_x g(s,X,V)}{\partial_{v_1} g(s,X,V)}\right)^2 \to +\infty$$

as $s \to T^-$.

It is known from [7] that the first singularity must occur at $v_1 = \pm 1$. Thus, we will choose initial conditions $g^{(0)}, B^{(0)}, E_2^{(0)}$ so that

$$\begin{aligned} &\left|\partial_x g^{(0)}(0,1)\right|, \left|\partial_{v_1} g^{(0)}(0,1)\right|, \frac{-\partial_x g^{(0)}(0,1)}{\partial_{v_1} g^{(0)}(0,1)}, \\ &\partial_x K(0,0) - \left(\frac{\partial_x g^{(0)}(0,1)}{\partial_{v_1} g^{(0)}(0,1)}\right)^2 \end{aligned} \tag{1.9}$$

are significantly larger than one. We'll take

$$E_2^{(0)} = B^{(0)}$$

so that, at least initially, the steep gradient in the field propogates with speed one. Also, note that by Maxwell's equations

$$\begin{aligned} &\frac{d}{dt}\int \left((\partial_x E_2)^2 + (\partial_x B)^2\right) dx\bigg|_{t=0} \\ = \quad &-2\int \partial_x E_2^{(0)} \partial_x j_2(0,x) dx \end{aligned}$$

so will also require

$$-\partial_x E_2^{(0)}(0) j_2(0,0) \tag{1.10}$$

to be significantly greater than one. Finally, we also require

$$K(0,0) \tag{1.11}$$

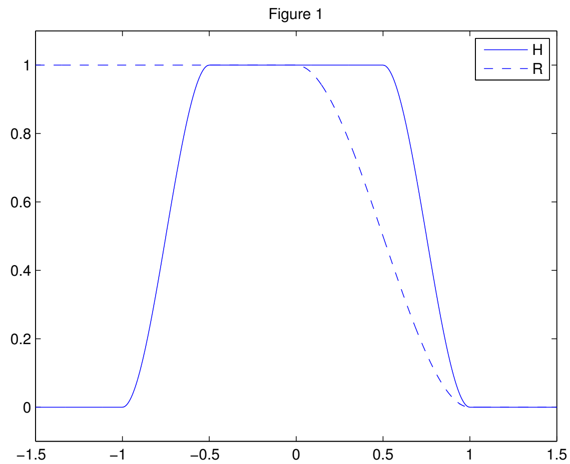
to be near zero so that $V_1(s,0,0,1)$ moves away from one slowly.

Define

$$R(u) = \begin{cases} 1 & \text{if } u \le 0 \\ (1+2u)(u-1)^2 & \text{if } 0 < u < 1 \\ 0 & \text{if } 1 \le u \end{cases}$$

and

$$H(w) = R(2|u| - 1).$$




Then for

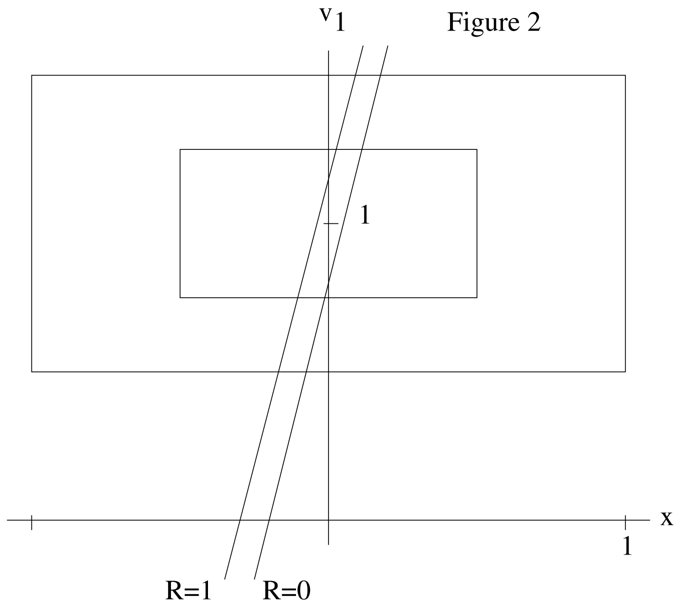
$$0 < \varepsilon < \frac{1}{2} \tag{1.12}$$

define

$$g^{(0)}(x, v_1) = H(x)H(2(v_1 - 1))R\left(\frac{x}{\varepsilon^2} + \frac{1}{2} - \frac{v_1 - 1}{\varepsilon}\right)$$

and

$$B^{(0)}(x) = E_2^{(0)}(x) = H(x)R\left(\frac{1}{2} + 1.5\frac{x}{\varepsilon^2}\right).$$




The condition (1.12) ensures that

$$\frac{x}{\varepsilon^2} + \frac{1}{2} - \frac{v_1 - 1}{\varepsilon} = 1 \text{ and } v_1 = \frac{3}{2} \Rightarrow x < \frac{1}{2}$$

and

$$\frac{x}{\varepsilon^2} + \frac{1}{2} - \frac{v_1 - 1}{\varepsilon} = 0 \text{ and } x = 0 \Rightarrow v_1 < \frac{5}{4},$$

as is depicted in Figure 2. For $|x| < \frac{1}{2}$ and $|v_1 - 1| < \frac{1}{4}$ we have

$$g^{(0)}(x, v_1) = R\left(\frac{x}{\varepsilon^2} + \frac{1}{2} - \frac{v_1 - 1}{\varepsilon}\right)$$

so

$$\begin{aligned} \partial_x g^{(0)}(0,1) &= \varepsilon^{-2} R'\left(\frac{1}{2}\right) &= -1.5\varepsilon^{-2}, \\ \partial_{v_1} g^{(0)}(0,1) &= -\varepsilon^{-1} R'\left(\frac{1}{2}\right) &= 1.5\varepsilon^{-1}, \end{aligned}$$

and

$$\frac{-\partial_x g^{(0)}(0,1)}{\partial_{v_1} g^{(0)}(0,1)} = \varepsilon^{-1}.$$

We'll take

$$w_0 = 0$$

and

$$b(x) = \frac{3}{4}H(2x - \frac{1}{4}.)$$

This choice of $b$ satisfies (1.2).

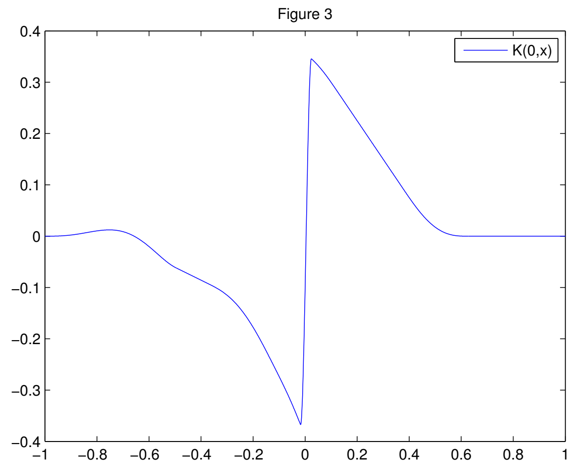
Now, omitting some elementary calculations,

$$\begin{aligned}
\partial_x K(0,0) &= \int g^{(0)}(0, v_1)dv_1 - b(0) \\
&\quad -(B^{(0)}(0))^2 - \int_{-\infty}^{0} B^{(0)}(\tilde{x})d\tilde{x}\frac{dB^{(0)}}{dx}(0) \\
&= \frac{3}{8} - \frac{3}{4} - \left(\frac{1}{2}\right)^2 \\
&\quad -\left(\frac{3}{4} - \frac{3}{32}(1.5\varepsilon^{-2})^{-1}\right)\left(1.5\varepsilon^{-2}\left(-\frac{3}{2}\right)\right) \\
&= \frac{27}{16}\varepsilon^{-2} - \frac{49}{64}.
\end{aligned}$$

Also, approximating we have,

$$\begin{aligned}
K(0,0) &= \int_{-\infty}^{0}\left(\int g^{(0)}dv_1 - b\right)dx - \int_{-\infty}^{0} B^{(0)}dx\, B^{(0)}(0) \\
&\approx \iint g^{(0)}dv_1\, dx - \int_{-\infty}^{0} b\, dx - \int_{-\infty}^{0} B^{(0)}dx\, B^{(0)}(0) \\
&= \frac{9}{16} - \frac{3}{16} - \left(\frac{3}{4} - \frac{3}{32}(1.5\varepsilon^{-2})^{-1}\right)\frac{1}{2} \\
&= \frac{\varepsilon^2}{32}.
\end{aligned}$$

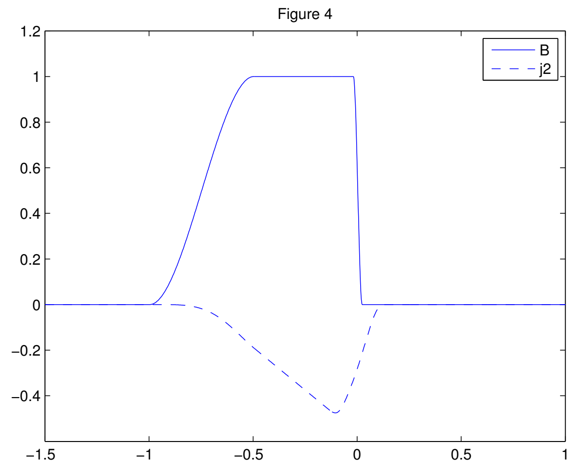
$K(0, x)$ is graphed in Figure 3 for $\varepsilon = \frac{1}{4}$.




Also

$$
\begin{aligned}
& -\partial_x E_2^{(0)}(0)\partial_x j_2(0,0) \\
= \ & -\partial_x B^{(0)}(0)\left(-B^{(0)}(0)\int g^{(0)}(0,v_1)dv_1 - \int_{-\infty}^{0} B^{(0)}(x)dx \int \partial_x g^{(0)}(0,v_1)dv_1\right) \\
= \ & \left(-\frac{3}{2}\varepsilon^{-2}\left(-\frac{3}{2}\right)\right)\left[-\frac{1}{2}\left(\frac{3}{8}\right) - \left(\frac{3}{4} - \frac{3}{32}\left(\frac{3}{2}\varepsilon^{-2}\right)^{-1}\right)(-\varepsilon^{-1})\right] \\
= \ & \frac{108\varepsilon^{-3} - 27\varepsilon^{-2} - 9\varepsilon^{-1}}{64}.
\end{aligned}
$$

$E_2^{(0)} = B^{(0)}$ and $j_2(0,x)$ are graphed in Figure 4 for $\varepsilon = \dfrac{1}{4}$.




Thus, the conditions (1.9), (1.10), and (1.11) are satisfied.

## 2 Numerical Methods

The problem (1.6) was solved using two different methods, a particle method and a finite difference approach introduced in [3] (see also [21]). For a complete discussion of particle methods see [1]. Let us outline the particle method used. Let $\Delta t = \Delta x > 0$ and $\Delta v > 0$ and denote

$$t^n = n\Delta t,\ x_k = k\Delta x,\ v_l = l\Delta v.$$

If

$$\begin{aligned} X^n_{k,l} &\approx X(t^n, 0, x_k, v_l) \\ V^n_{k,l} &\approx V_1(t^n, 0, x_k, v_l) \end{aligned}$$

then the basic approximation of the particle method is

$$g(t^n, x, v_1) \approx \sum_{k,l} q_{k,l}\, \delta_{\Delta x}(x - X^n_{k,l})\delta(v_1 - V^n_{k,l})$$

where

$$q_{k,l} = g^{(0)}(x_k, v_l)\Delta x\, \Delta v$$

and

$$\delta_{\Delta x}(x) = \begin{cases} (\Delta x)^{-1}\left(1 - \dfrac{|x|}{\Delta x}\right) & \text{if } |x| < \Delta x \\ 0 & \text{otherwise.} \end{cases}$$

If

$$A_i^n \approx A(t^n, x_i)$$

then we use

$$\begin{aligned} j_2(t^n, x_i) &\approx j_i^n \\ &= \sum_{k,l} q_{k,l} \delta_{\Delta x}(x_i - X_{k,l}^n)(w^0 - A_i^n). \end{aligned}$$

$A$ is computed using (with $\Delta t = \Delta x$)

$$\frac{A_k^{n+1} - 2A_k^n + A_k^{n-1}}{(\Delta t)^2} = \frac{A_{k+1}^n - 2A_k^n + A_{k-1}^n}{(\Delta x)^2} + j_k^n.$$

Then we take

$$B_{k+\frac{1}{2}}^n = \frac{A_{k+1}^n - A_k^n}{\Delta x}$$

and

$$E_{2,k}^{n+\frac{1}{2}} = -\frac{A_k^{n+1} - A_k^n}{\Delta t}.$$

Linear interpolation is performed on these values as needed. $E_{1k}^n$ is obtained by similar approximations of

$$\partial_x E_1 = \rho.$$

Then $X_{k,l}^n$ and $V_{k,l}^n$ are advanced by

$$\frac{X_{k,l}^{n+1} - X_{k,l}^n}{\Delta t} = V_{k,l}^{n+\frac{1}{2}}$$

and

$$\frac{V_{k,l}^{n+\frac{1}{2}} - V_{k,l}^{n-\frac{1}{2}}}{\Delta t} = \left(E_1^n + (w_0 - A^n)B^n\right)\Big|_{X_{k,l}^n} \tag{2.1}$$

where the right hand side of (2.1) is computed using linear interpolation.

The other method solves the Vlasov equation by making a half time step in $x$, a whole time step in $v$, and then another half time step in $x$. To advance $x$ from $t^n$ to $t^{n+\frac{1}{2}}$ let

$$\overline{X} = x_k - \frac{1}{2}\Delta t(l\Delta v)$$

and write

$$\overline{X} = x_{\bar{k}} + \theta \Delta x$$

with $\bar{k} \in \mathbb{Z}$ and $\theta \in [0, 1)$. Then define

$$\tilde{g}_{k,l} = (1-\theta) g^n_{\bar{k},l} + \theta g^n_{\bar{k}+1,l}.$$

Then the fields are computed at time level $t^{n+\frac{1}{2}}$ from $\tilde{g}$. The time step in $v$ is obtained by letting

$$\overline{V}_1 = (l\Delta v) - \Delta t \left( E^{n+\frac{1}{2}}_{1k} + (w_0 - A^{n+\frac{1}{2}}_k) B^{n+\frac{1}{2}}_k \right)$$

and

$$\overline{V}_1 = (\bar{l}\Delta v) + \theta \Delta v$$

with $\bar{l} \in \mathbb{Z}$ and $\theta \in [0, 1)$ and then taking

$$\tilde{\tilde{g}}_{k,l} = (1-\theta)\tilde{g}_{k,\bar{l}} + \theta \tilde{g}_{k,\bar{l}+1}.$$

Then $g^{n+1}$ is obtained from $\tilde{\tilde{g}}$ as $\tilde{g}$ was obtained from from $g^n$.

As we will see, the implementations of both methods produce results which converge to the same limits. Also, solutions of (1.6) conserve the energy:

$$\iint g(v_1^2 + (w_0 - A)^2) dv_1\, dx + \int (E_1^2 + E_2^2 + B^2) dx.$$

The results of both implementations produce only small variations in energy, smaller as $\Delta x \to 0$, $\Delta v \to 0$.

Also, the particle method was applied to the exact solution:

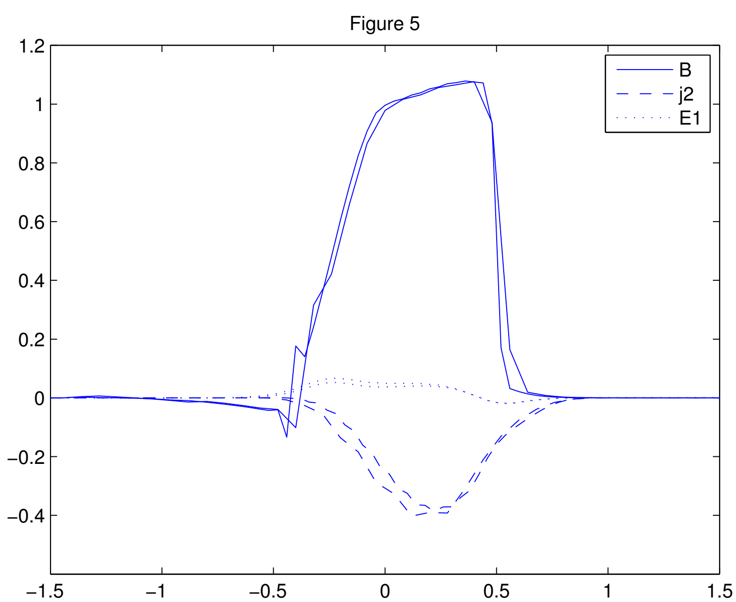
$$\begin{aligned}
E_1 &= -U' \\
U(x) &= \begin{cases} -(1-x^2)^3 & \text{if } |x| < 1 \\ 0 & \text{otherwise} \end{cases} \\
g &= \mathcal{G}\left(\frac{1}{2}v_1^2 + U(x)\right) \\
\mathcal{G}(e) &= \begin{cases} e^2 & \text{if } e < 0 \\ 0 & \text{if } 0 \le e \end{cases}
\end{aligned}$$

$$b(x) = \begin{cases} \dfrac{16\sqrt{2}}{15}(1-x^2)^{\frac{15}{2}} + (6 - 36x^2 + 30x^4) & \text{if } |x| < 1 \\ \\ 0 & \text{if } 1 \le |x| \end{cases}$$

$$A = E_2 = B = w_0 = 0.$$

The numerical results agreed well with the exact solution.

To exhibit the results stated above, consider solving (1.6) with the initial conditions described in Section 1 (with $\varepsilon = \frac{1}{4}$) to a final time of 0.48. Figure 5 displays the graphs of $B$, $j_2$, and $E_1$ at $t = .48$ computed with the difference scheme with $\Delta x = \Delta t = \Delta v = 0.08$ and $\Delta x = \Delta t = \Delta v = 0.04$. Figure 6 displays the corresponding results from the particle method. The relatively coarse mesh values of 0.08 and 0.04 were used so that the graphs from the two mesh values were visibly different. Figure 7 displays the graphs of $B$, $j_2$, $E_1$ at $t = 0.48$ computed with both the difference and particle methods with $\Delta x = \Delta t = \Delta v = 0.01$. The results of the two methods are hard to distinguish; as the mesh is refined further they cannot be distinguished. Table 1 gives the relative change ((final value-initial value)/initial value) of the computed energy over the interval $[0, 0.48]$ for both methods.



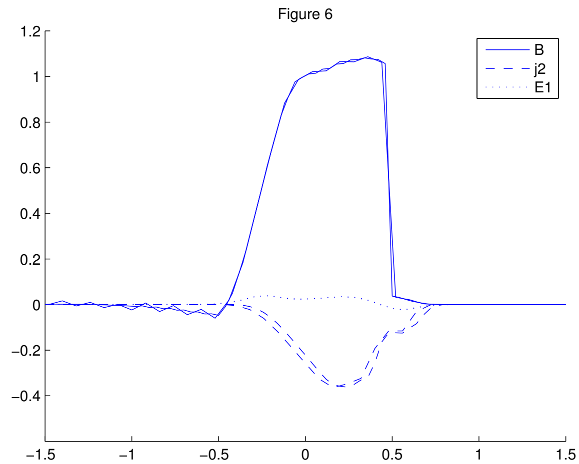


Figure 6
B
j2
E1

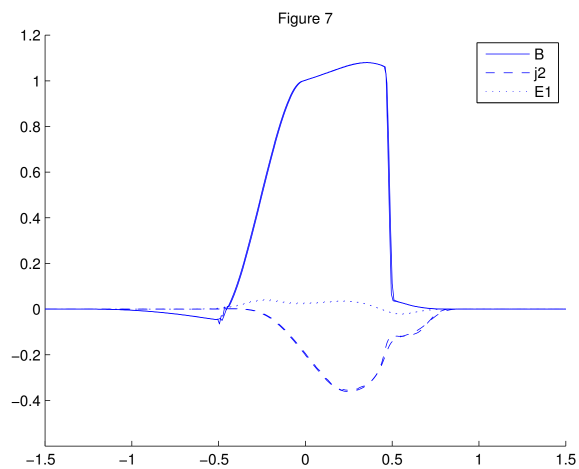



Figure 7
B
j2
E1

**Table 1**
**Relative Change in Energy**

| $\Delta x = \Delta v = \Delta t$ | Difference Method | Particle Method |
|---|---|---|
| .08 | $1.13 \quad 10^{-1}$ | $1.80 \quad 10^{-3}$ |
| .04 | $1.09 \quad 10^{-1}$ | $7.60 \quad 10^{-4}$ |
| .02 | $1.71 \quad 10^{-2}$ | $8.78 \quad 10^{-4}$ |
| .01 | $3.69 \quad 10^{-3}$ | $1.65 \quad 10^{-4}$ |
| .005 | $1.58 \quad 10^{-3}$ | $6.33 \quad 10^{-5}$ |
| .0025 | $7.79 \quad 10^{-4}$ | $2.92 \quad 10^{-5}$ |
| .00125 | $3.91 \quad 10^{-4}$ | $1.42 \quad 10^{-5}$ |

# 3 Computational Results

Now we present computational results for the initial conditions described before with $\varepsilon = \dfrac{1}{4}$. Figure 8 contains a plot of the level curves of $g$ at time zero (on the left) and of $g$ at time 0.4 (on the right). Figure 9 shows the graphs of $B, j_2, E_1$ as functions of $x$ at times $0, 0.2$ and $0.4$. For $B$ and $j_2$, later time produces a graph that is further to the right. For $E_1$ the maximal value decreases as $t$ increases. In Figure 10

$$\max_x |\partial_x E_2| \text{ and } \max_x |\partial_x B|$$

are graphed as functions of $t$ both for $\Delta x = \Delta v = \Delta t = 0.001$ and $\Delta x = \Delta v = \Delta t = 0.0005$. It should be noted that both differentiation of the fields and taking the maximal value are poorly behaved numerically and hence we do not expect the values from $\Delta x = \Delta v = \Delta t = 0.001$ and $\Delta x = \Delta v = \Delta t = 0.0005$ to match that closely. Still Figure 10 seems to indicate that the derivatives of $E_2$ and $B$ are growing with $t$ only modestly. We see from Figure 11 that

$$\left|\partial_x g\right| \text{ and } \left|\partial_{v_1} g\right|$$

are growing much more rapidly. Figure 11 contains the graphs of

$$\partial_x g(t, X(t, 0, 0, 1),\ V_1(t, 0, 0, 1))$$

and

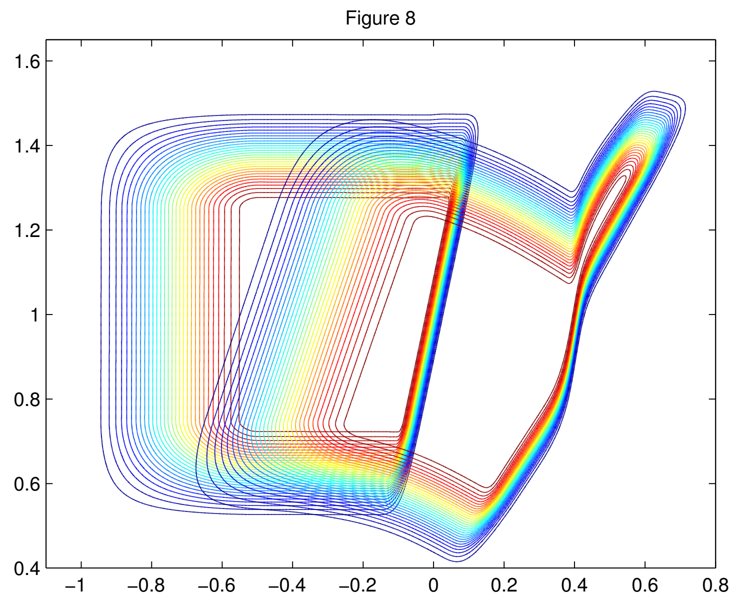
$$\partial_{v_1} g(t, X(t, 0, 0, 1),\ V_1(t, 0, 0, 1))$$

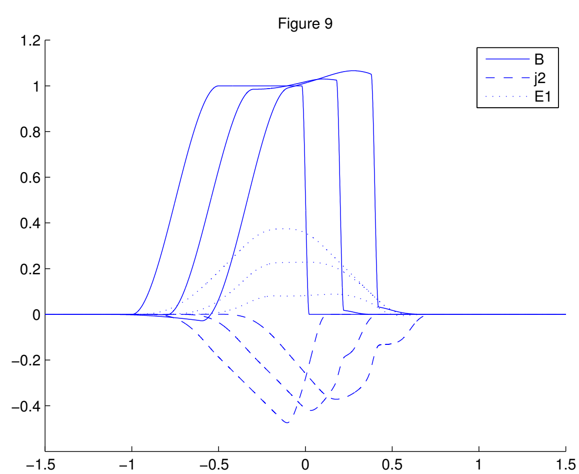
for both $\Delta x = \Delta v = \Delta t = 0.001$ and $0.0005$.

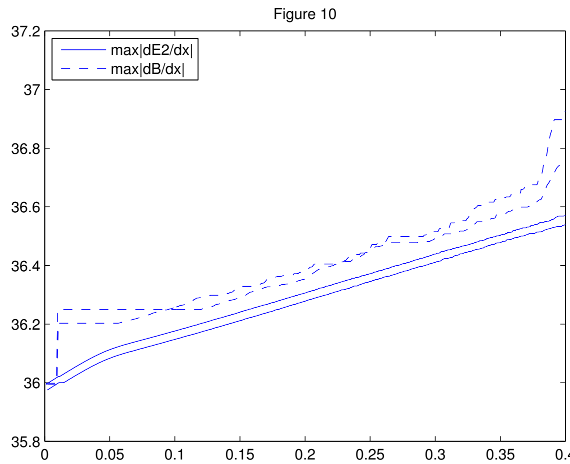
In conclusion, we find that despite going to lengths to choose initial conditions which generate a rapidly growing solution, the spatial derivatives of the fields do not grow rapidly. This is significant as these derivatives must be unbounded for the solution to blow up. Exponential growth of $|\partial_x g|$ and $|\partial_{v_1} g|$ does not imply blow up, for example, if $g$ satisfies the linear PDE

$$\partial_t g + v_1 \partial_x g + \partial_{v_1} g = 0,$$

then $g$ is as smooth as $g(0,\cdot,\cdot)$ but $|\partial_x g|$ and $|\partial_{v_1} g|$ grow exponentially.



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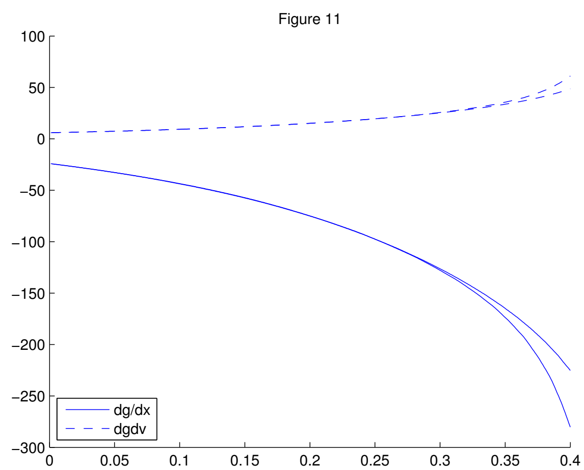


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