Quiz 7

Problem.

1. $\lim_{x\to\infty} x^{\frac{\ln(2)}{1+\ln(x)}}$
2. $\lim_{x\to 0} \csc(x) - \cot(x)$

Solution.

1. We begin by taking the logarithm, as this turns into an indeterminate form (namely ∞^0).

$$\begin{aligned}\lim_{x\to\infty} \ln(x^{\frac{\ln(2)}{1+\ln(x)}}) &= \lim_{x\to\infty} \frac{\ln(2)}{1+\ln(x)} \cdot \ln(x) \\ &= \ln(2) \cdot \lim_{x\to\infty} \frac{\ln(x)}{1+\ln(x)}\end{aligned}$$

We now can use L'Hôpital's Rule.

$$\begin{aligned}\ln(2) \cdot \lim_{x\to\infty} \frac{\ln(x)}{1+\ln(x)} &= \ln(2) \cdot \lim_{x\to\infty} \frac{\frac{1}{x}}{\frac{1}{x}} \\ &= \ln(2)\end{aligned}$$

Remember, you took a log. Therefore, the solution is $e^{\ln(2)}$, which is simply 2

2.

$$\begin{aligned}\lim_{x\to 0} \csc(x) - \cot(x) &= \lim_{x\to 0} \frac{1}{\sin(x)} - \frac{\cos(x)}{\sin(x)} \\ &= \lim_{x\to 0} \frac{1-\cos(x)}{\sin(x)} \\ &= \lim_{x\to 0} \frac{\sin(x)}{\cos(x)} \\ &= 0\end{aligned}$$