Given following function: -

$$
f(x)=-\cos (x)-\frac{x}{2}
$$

On the interval $[0,2 \pi]$ :

1. What interval(s) is the function increasing?
2. What interval(s) is the function decreasing?
3. Where is the function neither increasing nor decreasing?

Solution. Note

$$
f^{\prime}(x)=\sin (x)-\frac{1}{2}
$$

1. When the function is increasing, the derivative will be positive. Thus we need to solve when

$$
\sin (x)-\frac{1}{2}>0
$$

ie when

$$
\sin (x)>\frac{1}{2}
$$



From the picture (and from knowing when $\sin (x)=\frac{1}{2}$ ) one can see that this is true on the interval $\left(\frac{\pi}{6}, \frac{5 \pi}{6}\right)$.
2. We need to show when the derivative is negative. As you can see, it will be on the intervals $\left[0, \frac{\pi}{6}\right)$ and $\left(\frac{5 \pi}{6}, 2 \pi\right]$
3. This is when

$$
\sin (x)=\frac{1}{2}
$$

This happens at $x=\frac{\pi}{6}$ and $x=\frac{5 \pi}{6}$.

