Quiz 10

Problem.

1. $\frac{d}{du}\int_{-u}^{u} \arctan\left(\frac{1}{x}\right)\ dx$

2. $\frac{d}{dt}\int_{-t^2}^{1} \sqrt{4x^2+3}\ dx$

3. $\frac{d}{dv}\int_{4}^{v} \sin(3x^2)\ dx$

Solution. 1.

$$\begin{aligned}
\frac{d}{du}\int_{-u}^{u} \arctan\left(\frac{1}{x}\right) dx &= \frac{d}{du}\left(\int_{-u}^{0} \arctan\left(\frac{1}{x}\right) dx + \int_{0}^{u} \arctan\left(\frac{1}{x}\right) dx\right)\\
&= \frac{d}{du}\left(-\int_{0}^{-u} \arctan\left(\frac{1}{x}\right) dx + \int_{0}^{u} \arctan\left(\frac{1}{x}\right) dx\right)\\
&= -\arctan\left(-\frac{1}{u}\right)\cdot(-1) + \arctan\left(\frac{1}{u}\right)\\
&= \arctan\left(-\frac{1}{u}\right) + \arctan\left(\frac{1}{u}\right)\\
&= 0
\end{aligned}$$

2.

$$\begin{aligned}
\frac{d}{dt}\int_{-t^2}^{1} \sqrt{4x^2+3}\ dx &= \frac{d}{dt}\left(-\int_{1}^{-t^2} \sqrt{4x^2+3}\ dx\right)\\
&= -\sqrt{4(-t^2)^2+3}\cdot\left(\frac{d}{dt}(-t^2)\right)\\
&= -\sqrt{4t^4+3}\cdot(-2t)\\
&= 2t\sqrt{4t^4+3}
\end{aligned}$$

3.

$$\frac{d}{dv}\int_{4}^{v} \sin(3x^2)\ = \sin(3v^2)$$