

Analysis of a simplex tableau (ignoring degeneracy)

Is the corresponding basic feasible solution **optimal**?

- If there is a negative entry in the objective row: **No**, the corresponding basic feasible solution is not optimal. (Increasing the value of the corresponding nonbasic variable will increase the objective value.)

In this case: Is the linear program **unbounded**?

- If there exists a negative entry in the objective row having no positive entries below it: **Yes**, the linear program is unbounded. (The value of the corresponding nonbasic variable can be made arbitrarily large in such a way that no constraints are violated, and this will make the objective value arbitrarily large as well.)
 - If every negative entry in the objective row has at least one positive entry below it: **No conclusion**. (The objective value can be increased by pivoting in a column having a negative entry in the objective row. This will lead to a new tableau. Then analyze the new tableau.)
- If all entries in the objective row are nonnegative: **Yes**, the corresponding basic feasible solution is optimal. (No variable can be increased to increase the objective value.)

In this case: Is this optimal solution **unique**?

- If all nonbasic columns have positive (i.e., nonzero) entries in the objective row: **Yes**, the optimal solution is unique. (Bringing any nonbasic variable into the basis will decrease the objective value.)
- If there exists a nonbasic column with a zero in the objective row: **No**, the optimal solution is not unique. (The value of the corresponding variable can be increased to get a different optimal solution.)

In this case: Is there **another optimal basic solution**?

- If there exists a nonbasic column having a zero in the objective row and at least one positive number below it: **Yes**, there is another optimal basic solution. (Pivoting in this column will produce it.)
- If none of the nonbasic columns having zero in the objective row has a positive number below: **No**, there is no other optimal basic solution. (But there are still infinitely many optimal solutions—there is an infinite ray of optimal solutions extending from the optimal basic solution.)