IP formulation examples

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1. **Machine shop reopening.** A small machine shop is reopening after a fire had forced it to close for extensive repairs. The shop has three product lines: plates, gears, and housings. Each product line requires specialized equipment, and because of inactivity and possible damage all equipment must be serviced before it is used.

   The shop plans to open on a limited basis for the first two weeks, employing only three workers, each for 40 hours per week. It has available 2800 units of metal and can purchase additional metal for $2 per unit. The per-unit labor, metal, overhead costs, and selling prices for their products are shown below.

	Labor (minutes)	Metal (units)	Overhead (dollars)	Selling price (dollars)
Plates	10	4	6	24
Gears	30	1	9	32
Housings	20	6	8	30

   The existing backlog of orders for gears includes mostly orders for large quantities. Therefore, management does not believe that it would be useful to make gears during the first two weeks unless the shop can produce at least 200 of them.

   The servicing costs are $600 for the plate equipment, $900 for the gear equipment, and $700 for the housing equipment. The shop does not expect to use all equipment in the first two weeks.

   Management has $2000 remaining from its fire insurance settlement, and plans to spend that sum on the necessary service and possibly additional metal stock. The large backlog of orders that accumulated while the shop was closed indicates that they can sell any products they make. The overhead is charged against the selling price.

   Management's goal for the first two weeks is to maximize the profit so that they can afford to reopen full operations as quickly as possible.

2. **Utility connections.** Five major electrical consumers A, B, C, D, and E (e.g., manufacturing plants, hospitals, and housing developments) are to be added in a region served by three power plants X, Y, and Z. The connection costs to the new consumers (in millions of dollars) are given in the table below. The objective is to connect the new consumers to the generating plants in the most economical way possible.

	A	B	C	D	E
X	2	2	3	1	8
Y	3	7	2	6	4
Z	5	4	4	3	6

   The needs of the new consumers are 12, 10, 15, 16, and 15, respectively. The available capacities of the generating plants are 40, 32, and 30, respectively.

   Two of the new consumers, A and B, are hospitals. In order to lessen the possibility that both hospitals could be without power simultaneously, they cannot both be connected to the same power plant.

   Determine which new consumers should be connected to which power plant in order to minimize the connection costs.

3. **Product introduction.** Esquire Products will produce four new product lines in the next month. The respective per-unit profits on the lines are \$200, \$220, \$185, and \$190. They are basically testing the market and do not wish to produce more than 700 of any one line. The respective fixed start-up costs for the products are \$4000, \$5000, \$3000, and \$3500. Each item produced will require a part called an autorhombulator. The supplier of the autorhombulators charges according to the following schedule:

$$\begin{cases} \$50 & \text{ordering charge,} \\ \$9 & \text{each for the first 100 units,} \\ \$6 & \text{each for all additional units.} \end{cases}$$

Esquire has budgeted \$20,500 for the start-up costs and the purchase of the autorhombulators. Product lines 1 and 2 require a half hour of production time per item while lines 3 and 4 require 0.4 hour per item. There will be 800 hours of production time available during the month. How many of each line should they produce in order to maximize their profit?

4. **Boolean satisfiability.** Determine truth values for the Boolean variables x_1, x_2, and x_3 so that the propositional formula

$$(x_1 \vee \bar{x}_2) \wedge (\bar{x}_2 \vee \bar{x}_3) \wedge (x_1 \vee x_2 \vee \bar{x}_3) \wedge (\bar{x}_1 \vee x_3)$$

is satisfied (i.e., evaluates to true), or determine that the formula is unsatisfiable. Here $\wedge$ denotes AND, $\vee$ denotes OR, and $\bar{x}_i$ denotes NOT x_i.