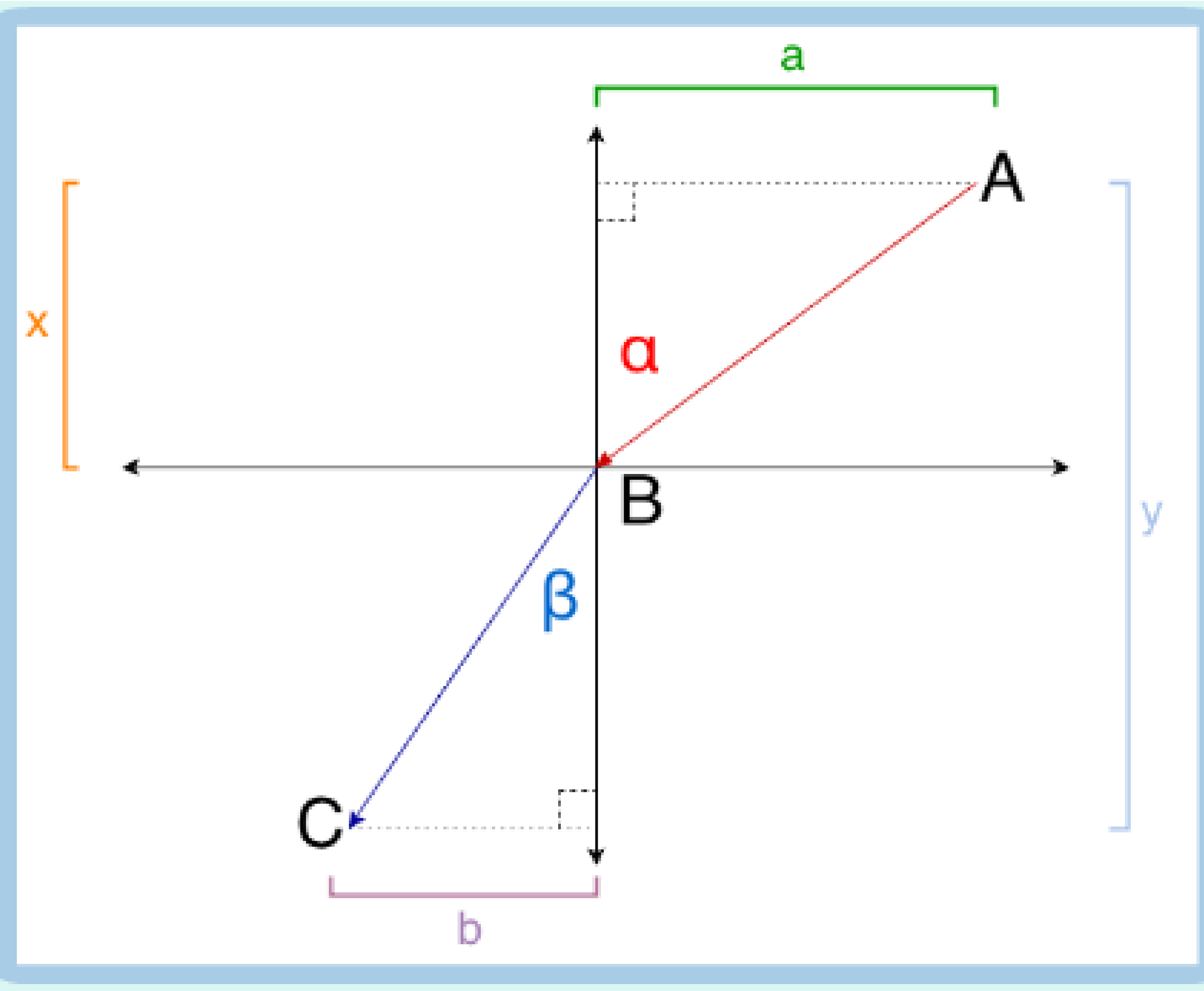
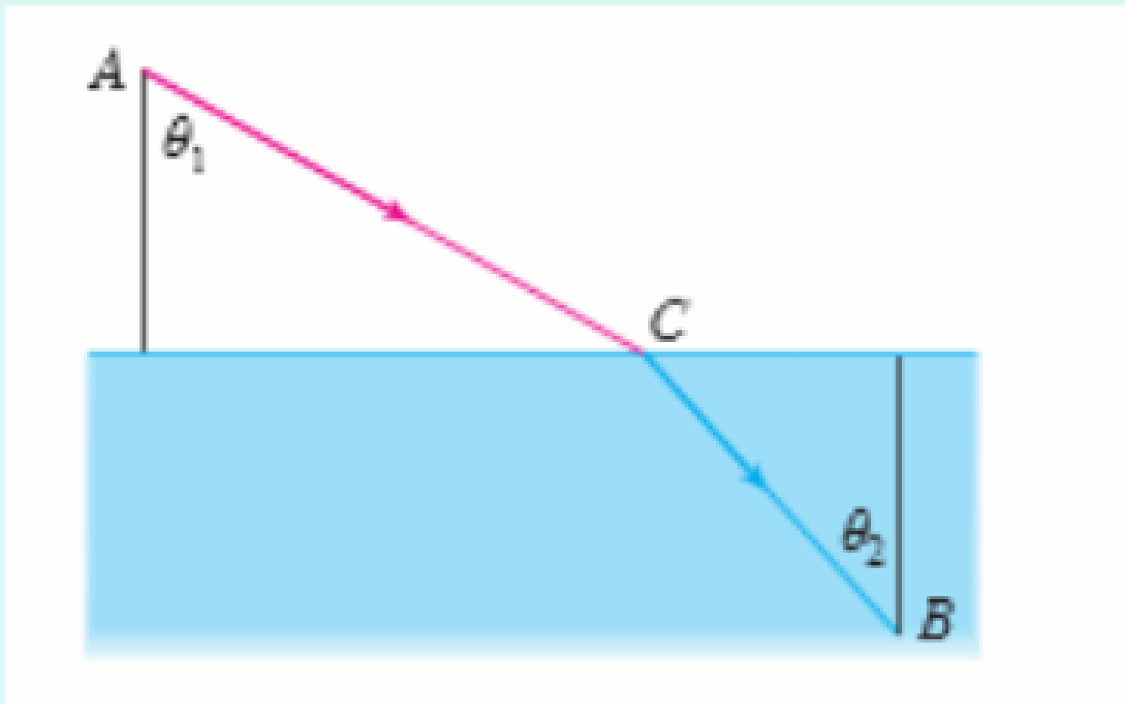
Snell's Law: n = c/v
where n is the refractive index of the medium,
c is the speed of light in vacuum,
and v is the speed of light in that medium.

sin1/sin2 = n2/n1 =v1/v2



Snell's Law



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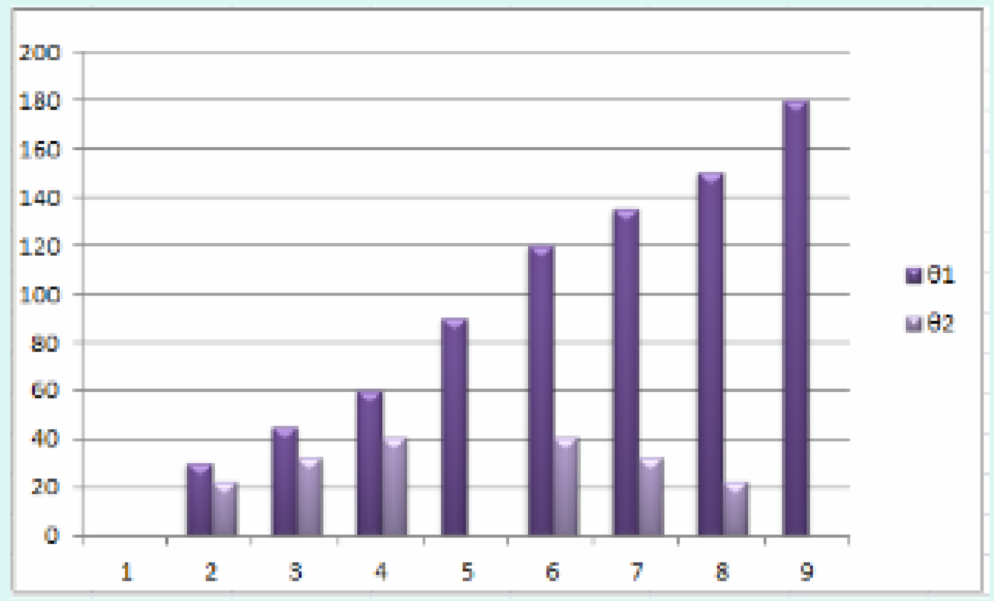
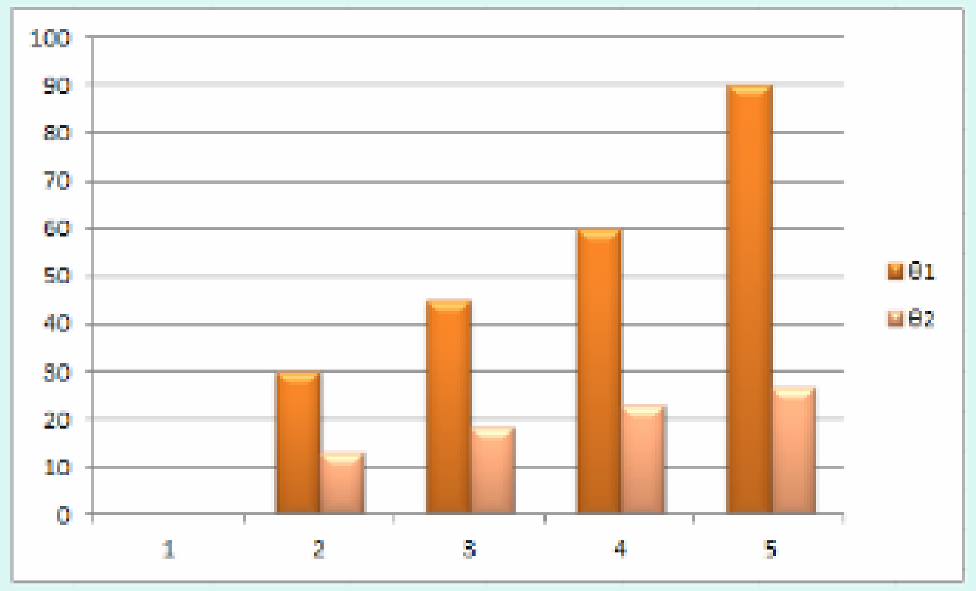
The first graph is for water while the second is when the ray hits the diamond surface.



theta 1 is the angle of incidence
theta 2 is the angle of refraction



As the index of refraction increases, the angle of deviation decreases (i.e. as theta 1 increases, theta 2 increases but with a smaller amount).

critical numbers when theta 1 = 0 , 90 and 180 since theta 2 will be either 0 or undefined

Distance = AB+BC = √(a^2+x^2) + √(b^2+(y-x)^2)
Velocity = distance/time
Time = distance/velocity = (√(a^2+x^2))/V1 + √(b^2+(y-x)^2)/V2
Let a, b and y be constants.
Let x be a variable. In order to minimize the time, we calculate the derivative with respect to x to be zero.
T = d/dx(√(a^2+x^2)/V1 + √(b^2+(y-x)^2)/V2)=0

$$\frac{d}{dx}\left(\frac{\sqrt{x^2+a^2}}{v_1}+\frac{\sqrt{b^2+(y-x)^2}}{v_2}\right)=0$$

$$\frac{x}{(v_1)\sqrt{x^2+a^2}}-\frac{y-x}{(v_2)\sqrt{b^2+(y-x)^2}}=0$$

$$sin\alpha=\frac{x}{\sqrt{x^2+a^2}}, \qquad sin\beta=\frac{-(y-x)}{\sqrt{b^2+(y-x)^2}}$$

$$\frac{sin\alpha}{v_1}-\frac{sin\beta}{v_2}=0$$

$$\frac{sin\alpha}{v_1}=\frac{sin\beta}{v_2}$$

$$\frac{sin\alpha}{sin\beta}=\frac{v_1}{v_2}$$

After we differentiate, we get that sin1/sin2=v1/v2=n2/n1.
This proves Fermat's Law which states that a ray of light travels in the least amount of time.

Second derivative test:

$$\frac{d^2T}{dx^2}=\frac{a^2}{v_1\sqrt{(a^2+x^2)^3}}+\frac{b^2}{v_2\sqrt{[b^2+(y-x)^2]^3}}$$

This last expression is positive for all values of x, so no matter what the critical number is, it is a local minimum.

Time is minimized at the critical value.