21-110: Problem Solving in Recreational Mathematics

Section A, spring 2010, 9 units

Syllabus

Time and place: Mondays, Wednesdays, and Fridays, 1:30–2:20 p.m., in Porter Hall A22

Textbooks: *Problem Solving Through Recreational Mathematics*, by Bonnie Averbach and Orin Chein, ISBN 0-486-40917-1; *Thinking Mathematically*, by John Mason, Leone Burton, and Kaye Stacey, ISBN 0-201-10238-2

Web page: `http://www.math.cmu.edu/~bkell/21110-2010s/`

Instructor: Brian Kell

Office: Physical Plant Building (PPB) 342. To get there: Go to Wean Hall. Take the elevator to the first floor. Exit Wean Hall (you will be facing north). Follow the sidewalk across the street and around the corner of the next building you come to (the FMS Building). You will come to a small plaza. At the southwest corner of this plaza is a small building with four big water chillers on the roof (or some kind of steam-producing things, anyway). That's the Physical Plant Building. My office is just inside the door.

Office phone: 412-268-1447

E-mail: `bkell@cmu.edu`

Office hours: Mondays and Wednesdays, 2:30–4:00 p.m.

If you want to meet at a different time, that's fine; please e-mail me to let me know. The most up-to-date version of my weekly schedule is available online at

`http://www.zifyoip.com/me/sched/2010/spring.html`

and is posted outside my office door; I am often in my office when I'm not doing something else.

Math department office: Wean Hall 6113, 412-268-2545

Welcome to Problem Solving in Recreational Mathematics!

The goals of this course are threefold:

- to develop problem-solving techniques that can be used to tackle various mathematical and logical problems;
- to improve your abilities to organize and analyze information, to think creatively about a problem, and to clearly communicate and justify your ideas; and
- to investigate some of the areas of recreational mathematics in the spirit of pure curiosity.

I welcome comments and feedback about this course at any time. Please feel free to send me an e-mail with your thoughts, stick a note under my office door, or deliver a note to the math department office and ask them to put it in my mailbox. For totally anonymous comments, visit the course Web page; there is a link to a form you can fill out that will send your comments to me anonymously.

Academic integrity

The most important rule in this course is this: You are expected to be honest. This is a simple matter of respect toward me and the other students in the class. You must abide by the Carnegie Mellon Statement on Academic Integrity, available online at `http://www.studentaffairs.cmu.edu/theword/acad_standards/integrity.html`. In this course, this primarily means that you should never turn in work that is not your own.

This does not mean that you are not allowed to work with other students. You are certainly welcome to collaborate on homework problems; it is often more fruitful and enjoyable to work with other people when trying to figure something out. They can give you a different perspective or fresh insight on the problem. Conversely, explaining one of your ideas to another problem forces you to clarify your thoughts and can help to highlight flaws you may have previously overlooked. However, if you work with others to come up with a solution, afterward you should go away and write it up on your own. You should not base your written homework on another student's written homework, and never put your name on anything you do not understand.

Please familiarize yourself with the Policy on Cheating and Plagiarism at `http://www.cmu.edu/policies/documents/Cheating.html` and the Undergraduate Academic Disciplinary Actions Overview at `http://www.cmu.edu/policies/documents/AcadRegs.html`. I will be happy to answer any questions you may have about academic integrity.

What is "recreational mathematics," anyway?

Some people think of mathematics as the process of adding and multiplying lots of numbers together. Others see it as a big collection of rules and formulas that need to be memorized and regurgitated when necessary to find "The Answer." There are also people who view mathematics merely as a tool that must be used in order to get useful things done in science, engineering, medicine, and so on.

While all of these impressions of mathematics are valid to a greater or lesser extent, there is another side of mathematics that most people do not see. This is the side of mathematics that is beautiful and intriguing in its own right. Many mathematicians study mathematics for its own sake—they think about mathematical problems and puzzles purely out of curiosity, not because these problems are particularly *useful* for anything. (On the other hand, again and again throughout history mathematical problems that were originally studied with no "useful" purpose in mind have found very important practical uses later. Why this should be so is a topic of much philosophical debate.)

This is what recreational mathematics is—mathematical problems that people play with for no other reason than that they are interesting. Topics in recreational mathematics include magic squares, chessboard problems, logical puzzles, paradoxes, Sudoku, Pythagorean triples, polyominoes, shapes and symmetry, origami, fractals, Pascal's triangle, probability, prime numbers, number bases, Fibonacci numbers, palindromes, Nim, knots, and even coloring shapes on the surface of a donut! We will look at some of these topics in this course.

In recreational mathematics the *answer* is not the important point. Much more important is the *process* used to find the answer. For one thing, the process can often be reused to solve other problems. Also, careful examination of the process not only can provide a proof that the answer is correct but also can lead to the discovery of deeper mathematical truths. For this reason, most of the points on homework assignments will be given for clear and thorough explanations and justifications of the reasoning underlying your solution.

The problems and puzzles in this class should be interesting to you. Hopefully you will find yourself thinking about these problems at odd times: in bed before falling asleep, in the shower, or walking around campus. That's a good sign!

Necessary background

There are no official prerequisites for 21-110. We will be using ideas from high-school algebra and geometry; if at any time you feel shaky with some of these ideas, please come talk to me and I will be glad to help refresh your memory. I doubt we will use much trigonometry, and there will be no calculus knowledge required.

Class structure

Usually at the end of class I will give a reading assignment or a problem to work on before the next class. If I give a problem, I would like at least one person to present a solution to it at the beginning of the next class. Everyone should aim to present two or three such solutions over the course of the semester.

As far as possible, I would like to avoid standing at the front of the classroom and just delivering a lecture (though I can do this if the class would prefer it). I think that the course will be more interesting and enjoyable if we can have meaningful and lively discussions. However, this requires you to take this course seriously, do the assigned reading, and think about problems outside of class. If everyone does this, then when class starts everyone will have a basic idea of what's going on, and we can spend more time exploring the details and tackling some of the more interesting and challenging ideas.

Grading

There will be roughly 10 graded homework assignments given over the course of the semester, all of equal weight. Together these homework assignments will constitute 70% of your course grade. Another 20% will be a project. The remaining 10% of your course grade will be for class participation. Your final grade for this course will be determined according to the following scale.

If you get at least this percentage:	92%	84%	76%	68%
Then you are guaranteed at least this grade:	A	B	C	D

Graded homework

You should start thinking about the homework problems early. Most of the problems will require time, so that you can think about them from several different angles. It is not wise to wait until the night before the homework is due to start—this will lead to frustration. If you start early you can stay rested and *have fun* with the problems!

As described in the "Academic integrity" section above, you are welcome to work with other students on the homework, but write up your solution yourself. If you are really stuck on a homework problem, put it aside and come back to it later. If you still can't make progress, feel free to come talk to me about it.

Explaining your solutions will require a lot of writing. Often English sentences and paragraphs will constitute much more of your solution than mathematical symbols will (though symbols have their place too, of course). For this reason I recommend typing your solutions (except perhaps figures), so that you can easily and cleanly revise what you have written. The homework you hand in should be a polished final draft, but it will take several intermediate drafts to get there. Do not expect to produce your final draft on your first attempt!

Your submitted homework assignment is expected to meet the guidelines described in the "Quality expectations for submitted work" below.

Project

There will be a project in this course. You may work in a group of up to four people on the project. It will tie together several topics we will talk about in the course and will require creative thinking to complete. More information will be given about the project later.

Attendance and participation

Many math courses focus on technical mastery of a relatively fixed set of facts, techniques, and concepts. However, this course is somewhat different. It focuses primarily on the exploration and interpretation of mathematical ideas and problem-solving techniques, rather than the memorization and manipulation of formulas. An important part of this course will be group discussion and activities in class, so attendance and participation are strongly encouraged.

The class participation portion of your course grade will be based on such things as presenting problems at the beginning of class (see "Class structure" below), answering (and asking!) questions in class, contributing to class discussions, and so forth.

Just as it is important that you attend class, it is also important that you arrive on time. Coming in late is disruptive and disrespectful to me and your fellow classmates. If you do arrive late, please enter the room and take your seat quietly. Likewise, if you must leave class early, please do so as quietly and considerately as you can.

While in class, please turn off your cell phone or set it to silent mode.

If you miss class, you can check the calendar on the course Web page to find out what we did.

Quality expectations for submitted work

Submitted work (including the project) is expected to meet the following standards for quality. Work which does not meet these standards will not be accepted.

- **Be academically honest.** Never put your name on work that is not yours or work you do not understand, and always give credit where credit is due. Remember: Taking someone else's work or ideas and presenting them as your own is plagiarism, even if you change the wording. Provide a list of sources other than the textbooks (if any) that you used to do the assignment, and state clearly that you are copying or mimicking an example from the book (if appropriate). If you worked with other students in the class to solve a problem, be sure to write your results *in your own words*, and include a list of your collaborators. Do not ever copy text directly from another source, even with attribution, unless it is a short quote properly marked as such.
- Put your name (first and last) on the top of every page you turn in. Do not include your student ID number or your Social Security number. On the first page, include a short description of the assignment (a few words as a title would be fine—just so I know what it is I'm looking at).
- Leave wide margins (1.5 inches all around is nice), so that I have enough blank space on each page to write comments. Do not cram your work together. If you cannot fit all of your solution on one page, please continue onto another sheet of paper.
- Use good quality paper, write or print on only one side, and staple the pages together. If your work is typed, use a standard font and print in black. If your work is handwritten, use a pencil or blue or black ink. Do not hand in paper that has a fringe from being torn out of a notebook.
- Include page numbers, in case the pages are accidentally stapled in the wrong order.

How to read the textbooks

The textbooks are not meant to be read quickly, but slowly, carefully, and thoughtfully. Both have many problems scattered throughout the text, and at certain points they ask you to try the problems. When you come to one of these points, *stop*. Try to figure it out. Spend some time thinking about it. It may take you 15 or 20 minutes, or maybe longer. Don't continue reading until you have solved the problem, or you are sure you are stuck and can't get any further. The books are written to guide you through the process of problem-solving, but this guidance will be hollow unless you are actually attempting to solve problems!

Resources

In this room are many excellent resources for you: the other students. Working with other students will help you and help the others.

I am also very happy to help you. As a math instructor, my goal—my job—is to see you excel in math! Because time is limited, it may be difficult or impossible to answer all of your questions during class itself. By all means, please feel free to come talk to me if you have unanswered questions, if something really isn't making sense to you, or if you just would like to talk in greater detail about a topic we have covered. The best way to contact me is by e-mail, but you are welcome to stop by my office at any time. Of course, I cannot guarantee I will be available outside my office hours unless you have made an appointment.

Accommodations

Some students qualify for special accommodations due to various needs. If you are such a student, please present documentation supporting such a request as soon as possible, and I will do my best to assist you.